

# Macroeconomic Factors in Oil Futures Markets

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## Abstract

This paper constructs a macro-finance model for commodity futures and documents a new empirical fact that real economic activity forecasts oil futures returns and prices. The model generalizes previous futures pricing models and indicates significant predictable variation in oil risk premia which covaries strongly with real economic activity. The estimates reveal new dynamics between economic activity and oil futures, which are material to the valuation of real options.

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# 1 Introduction

Research in finance has yielded models that closely fit the dynamics of commodity futures markets. However, these models explain futures prices and risk premia in terms of purely latent state variables.<sup>1</sup> On the other hand, research in macroeconomics investigates the time series dynamics of oil prices with economic data, finding that oil shocks forecast recessions.<sup>2</sup> However, these studies exclusively examine the spot price of oil. Oil futures markets are much more active than spot markets, and possess a time-varying term structure which reflects both oil risk premia and the market's forecast of spot prices.

This paper constructs a futures pricing model with both latent and observed state variables which makes two main contributions to the literature. First, it allows one to estimate the joint dynamics of macroeconomic factors with futures prices and returns. To my knowledge this is the first canonical macro-finance model for futures markets. Second, it extends and generalizes previous models in a way that is key to the empirical findings. While previous models do not explicitly examine how futures risk premia interact with macroeconomic factors, they implicitly impose strong restrictions on those interactions. Specifically, they assume that all relevant information is fully reflected in the term structure of futures prices (perfect spanning). I find this restriction is rejected in the data.

Specifically, I document that measures of real economic activity forecast oil futures prices and returns, conditional on contemporaneous futures prices. Previous research shows that oil shocks affect the real economy, while this paper is the first to show a converse effect flowing from the real economy to the oil futures market and to examine the resulting feedback relation between real activity and oil.

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<sup>1</sup>e.g. [Gibson and Schwartz \(1990\)](#); [Schwartz \(1997\)](#); [Casassus and Collin-Dufresne \(2005\)](#)

<sup>2</sup>e.g. [Hamilton \(1983\)](#); [Bernanke et al. \(1997\)](#); [Hamilton \(2003\)](#); [Barsky and Kilian \(2004\)](#); [Kilian \(2009\)](#)

Studies such as [Casassus and Collin-Dufresne \(2005\)](#) and [Hamilton and Wu \(2014\)](#) suggest that the oil risk premium does not covary with the business cycle. This conclusion is a consequence of the spanning assumption plus the fact that the contemporaneous correlation of oil prices with the business cycle is low. Relaxing this restriction produces an estimated oil risk premium that better fits the historical returns to oil futures, more than tripling the  $R^2$  from 1.3% to 4.3% after adjusting for the additional degrees of freedom. The oil risk premium implied by the macro-finance model is nine times more volatile than that implied by previous models, and covaries strongly with the business cycle. These results indicate that previous models fail to capture the majority of variation in oil risk premia.

The model and estimates presented here make use of the full panel of oil futures prices and macroeconomic data, and reveal rich dynamics that are not available from time-series vector autoregressions. For example, it is well-documented that a higher oil price forecasts lower economic activity ([Hamilton, 1983](#); [Bernanke et al., 1997](#); [Kilian, 2009](#)). I find that the strength of this effect varies depending on the market's forecast of oil prices: oil shocks that are forecast by the market to be more persistent have a larger and longer lasting effect on real activity. Conversely, although shocks to real activity dissipate in less than a year the market forecasts that the resulting higher oil price persists for decades, perhaps because oil is a nonrenewable resource. I also find that the effects of oil shocks are stronger when the economy is in expansion, and that real activity drives oil prices through shocks to industrial production while oil shocks drive real activity through changes in consumer spending.

The findings in this paper contribute to our understanding of the joint dynamics of oil futures markets with the macroeconomy. Oil is the single most important commodity in the modern economy as reflected by its input share, trading volume, media coverage and academic and industry attention. The Energy Information Administration (EIA) estimates

that in 2010 expenditures on energy, the majority of which was petroleum-based, accounted for 8.3% of U.S. GDP.<sup>3</sup> On the financial side, oil is also the most important commodity futures market. Crude oil makes up 47% of the benchmark Goldman Sachs Commodity Index (GSCI) and refined oil products another 21%, compared to 3.5% for wheat and 2.8% for gold. In 2014 the average daily trading volume in Nymex crude oil futures was \$53 billion compared to an average daily trading volume of \$41 billion on the entire New York Stock Exchange (NYSE). The model and findings in this paper also contribute to our understanding of time-variation in oil risk premia, and thus contribute to this important literature as well.

Forward-looking measures of real activity are the most common macro factors in macro-finance studies of other markets<sup>4</sup> and are naturally material to the oil price forecast given oil's importance in the real economy. With respect to risk premia, previous studies have documented that real activity forecasts countercyclical returns in equity and bond markets. The discovery that real activity forecasts procyclical returns to oil futures echoes those findings, but in the opposite direction. I argue this finding is consistent with oil futures as a hedge asset for consumers, while I do not find support for the classic hedging pressure hypothesis.

[Pindyck \(1993\)](#) argues that the value of real options should depend on macro in addition to financial factors. However, this has not been emphasized in the literature on commodity real options (i.e. optimal resource extraction) because of the assumption of perfect risk spanning, which rules out the relevance of macro factors. In a calibrated example, I find that adding unspanned real activity to the model increases the value of a hypothetical oil well. When the payoff to a real option depends on a macro factor there are two channels by

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<sup>3</sup>The EIA's figure reflects expenditures on combustible petroleum products and does not include products such as asphalt, tar, wax, coke, lubricants, and petrochemicals that are critical inputs into many industries.

<sup>4</sup> See, e.g., [Chen, Roll and Ross \(1986\)](#), [Cooper and Priestley \(2008\)](#), [Ludvigson and Ng \(2009\)](#), [Joslin, Priebsch and Singleton \(2014\)](#).

which the macro factor affects option value: its unspanned dynamics and its risk premium. For reasonable parameter values I find that the dynamics effect dominates while the risk premium effect is very small.

## 1.1 Related Literature

There are two strands of the literature in commodity futures that this paper builds upon. In the first, commodity futures prices are modeled as affine functions of latent (unobserved) state variables. Classic examples are [Gibson and Schwartz \(1990\)](#), [Schwartz \(1997\)](#), and [Casassus and Collin-Dufresne \(2005\)](#); more recent examples include [Casassus, Liu and Tang \(2013\)](#) and [Hamilton and Wu \(2014\)](#). Models of this type do not incorporate macroeconomic data and the latent variables can often be rotated and translated without changing the likelihood ([Dai and Singleton \(2000\)](#); [Collin-Dufresne, Goldstein and Jones \(2008\)](#)), so their economic meaning is unclear. More subtly, they implicitly assume that all relevant information in the economy is reflected (spanned) in current futures prices and no other information can contribute incremental forecasting power. I find that real economic activity has material effects on risk premiums and forecasts of oil futures prices, over and above the information in current futures prices.

The second strand uses vector autoregressions (VARs) to explore the time series relation of oil prices with the real economy; examples include [Hamilton \(1983\)](#); [Hamilton \(2003\)](#); [Kilian \(2009\)](#); [Alquist and Kilian \(2010\)](#); [Kilian and Vega \(2011\)](#). These studies include a single state variable based on the spot price of oil. This approach does not incorporate the full panel of futures prices of different maturities, and is silent regarding risk premia and the market's forecast of the spot price. The model in this paper imposes the additional assumption that risk premiums are “essentially affine” ([Duffee, 2002](#)) in the state variables,

which lets us bring the full term structure of futures prices to bear on returns, forecasts, and risk spanning.

Fama and French (1987), Bessembinder and Chan (1992), Khan, Khoker and Simin (2008), Singleton (2013) and Hamilton and Wu (2015) run return forecasting regressions for individual returns to futures on individual commodities; Gorton, Hayashi and Rouwenhorst (2013) sort the cross section of commodities into portfolios. Szymanowska et al. (2013) decompose individual futures returns into a spot premium and a term premium. Hong and Yogo (2012) find that futures market open interest has robust predictive power for futures returns and bond returns. This paper contributes to this literature as well, as it offers a simple and consistent way to make use of the full term structure of futures prices and returns.

Chiang, Hughen and Sagi (2015) extract spanned factors from oil futures and a volatility factor from oil options, and find that exposure to the volatility factor carries a risk premium in equity markets but not in oil futures. In contrast, I examine the effects of the real economy on price forecasts and risk premia in oil futures markets, and the resulting dynamic feedback relationship, in a macro-finance setting.

## 2 Data and Forecasting Regressions

In this section I describe the data and investigate to what extent the macroeconomic time series are spanned by oil futures prices. The distinction between spanned and unspanned macro factors drives the modelling strategy. To see the distinction, let  $R_{t+j}$  be the payoff from going long a  $j$ -period futures contract at price  $F_t^j$  and holding it to maturity:

$$R_{t+j} = S_{t+j} - F_t^j$$

where  $S_{t+j}$  is the spot price at maturity. This accounting identity always holds ex post and thus holds in expectation for any information set  $X_t$  :

$$F_t^j = E[S_{t+j}|X_t] - E[R_{t+j}|X_t]$$

This is the case for the spanning assumption: any information relevant to forecasting spot prices or returns should be reflected in contemporaneous futures prices. However, the same argument applies to bond yields and bond returns, yet recent research finds evidence against it in bond markets. In particular, real activity ([Joslin, Priebsch and Singleton \(2014\)](#)) or latent factors extracted by filtering ([Duffee \(2011\)](#)) or from forecasts ([Chernov and Mueller \(2012\)](#)) help to forecast bond returns over and above information in the term structure of bond yields. The first contribution of this paper is to document similar evidence against the spanning assumption in oil futures markets.

## 2.1 Futures Price Data

I use closing prices for West Texas Intermediate (WTI) oil futures with maturities of one to twelve months, on the last business day of each month from January 1986 to June 2014. The futures price data is denoted

$$f_t^j = \log(F_t^j), j = 1 \dots J, t = 1 \dots T$$

$$f_t = \left[ f_t^1 \quad f_t^2 \quad \dots \quad f_t^J \right]'$$

where  $F_t^j$  is the closing price at end of month  $t$  of the future that expires in month  $t + j$ ,  $t = 1$  corresponds to 1/1986,  $T = 342$  corresponds to 6/2014, and  $J = 12$ . The maximum maturity of twelve months is because longer dated futures were seldom traded in the early years of the sample. The results do not change significantly if I extend  $J$  to, e.g., 24 months maturity.

## 2.2 Macro Factors

I use the Chicago Fed National Activity Index (CFNAI), hereafter labelled  $GRO$ , as a forward-looking measure of real economic activity. The CFNAI is a weighted average of 85 U.S. macroeconomic time series, published monthly by the Chicago Fed.<sup>5</sup>

Measures of real activity have been found to forecast inflation (Stock and Watson, 1999), bond returns (Ludvigson and Ng, 2009) and equity returns (Cooper and Priestley (2008)) and are commonly used in macro-finance studies of the term structure (Ang and Piazzesi 2003; Joslin, Pribsch and Singleton 2014). Importantly, the results in this paper do not depend on using the CFNAI measure specifically but also obtain using other indexes of real activity (see Appendix A).

The second macro factor is the inventory of oil in readily available storage. The Theory of Storage (Working (1949)) predicts a natural equilibrium between the cost of carry in the futures market and the level of inventories held. Also, Gorton, Hayashi and Rouwenhorst (2013) find that inventories are associated with returns to futures contracts, using portfolio sorts of the cross-section of commodity futures. I use the log of the Energy Information Administration’s “Total Stocks of Commercial Crude Oil excluding the Strategic Petroleum Reserve” as a measure of the available inventory of crude oil,<sup>6</sup> hereafter labelled  $INV$ .

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<sup>5</sup><https://www.chicagofed.org/publications/cfnai/index>

<sup>6</sup><http://www.eia.gov/petroleum/>



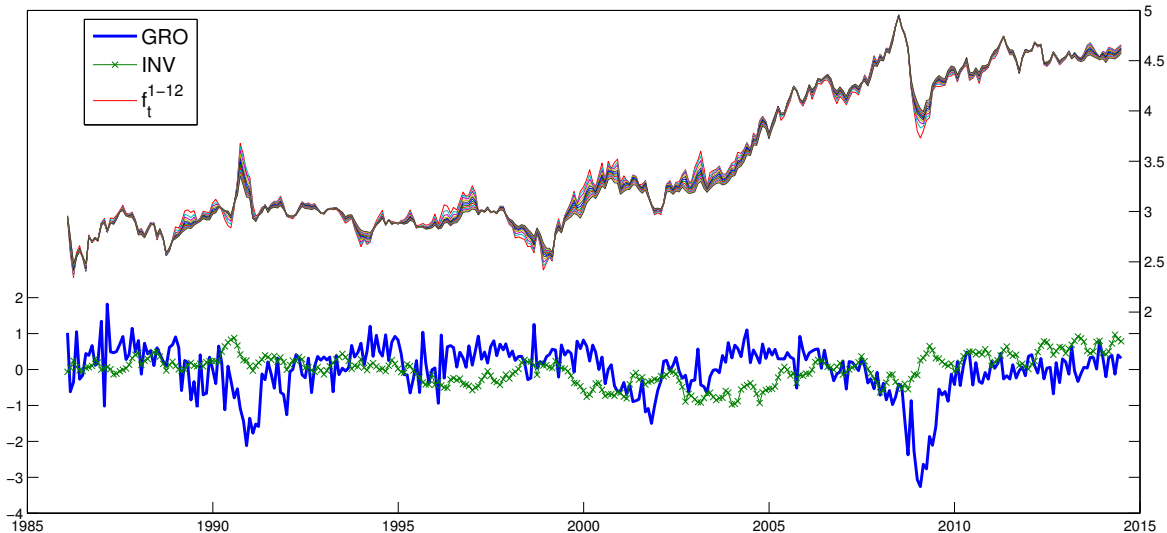


Figure 1: The figure plots log futures prices for Nymex crude oil  $f_t^{1-12}$ , the Chicago Fed National Activity Index  $GRO$ , and the log of the EIA’s monthly U.S. oil inventory  $INV$  from January 1986 to June 2014.

Thus, the macro factors are  $M_t = [GRO_t, INV_t]'$ . Figure 1 plots the time series of log oil futures prices and the macro factors  $GRO$  and  $INV$ .

### 2.3 Stylized Facts

Previous affine futures pricing models (e.g. [Gibson and Schwartz \(1990\)](#); [Schwartz \(1997\)](#); [Casassus and Collin-Dufresne \(2005\)](#)) assume that all relevant factors are spanned by the futures market and hence they are estimated using futures prices only. As [Duffee \(2011\)](#) and [Joslin, Le and Singleton \(2013\)](#) observe in the context of bond yields, the spanning assumption has strong implications for the joint behavior of futures prices and the economy. First, any such model with  $N$  state variables can be rotated into “reduced form” such that the reduced-form state variables are equal to the prices of  $N$  arbitrary linearly independent portfolios of futures contracts ([Duffie and Kan \(1996\)](#)). Second, the  $N$  portfolios explain log

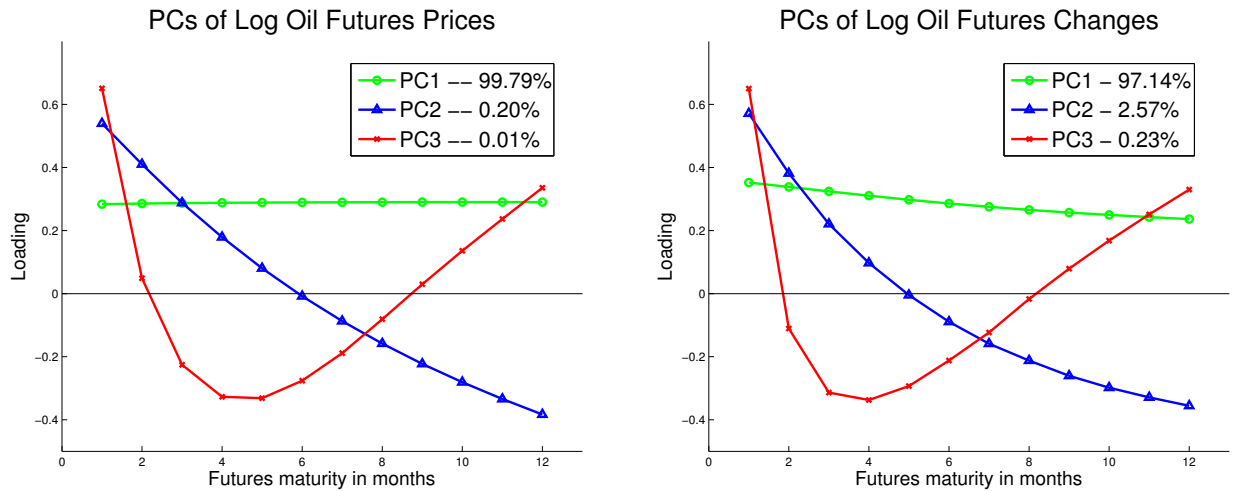


Figure 2: Loadings of the first three principal components (PCs) of the levels (panel A) and changes (panel B) of log oil futures prices, monthly from 1/1986 - 7/2014. The legend displays the fraction of total variance explained by the PCs.

futures prices up to idiosyncratic errors. Third, conditional on the prices of the  $N$  portfolios, no other information can contribute incremental forecasting power.

I document three stylized facts that contradict these restrictions. First, oil futures prices and returns display a low dimensional factor structure. Second, most variation in the macro factors  $M_t$  – in particular the real activity factor  $GRO$  – is not spanned by variation in oil futures prices. Third, and most important, I find that  $M_t$  contributes incremental forecasting power for oil prices and returns over and above the information in contemporaneous futures prices.

### A) Oil futures prices display a low dimensional factor structure

Figure 2 plots the loadings of the first three principal components (PCs) for the levels and changes in log oil futures prices by maturity. The first two PCs have the familiar level and slope patterns of loadings, and account for 99.9% of the variation in log price levels and

99.7% of variation in log price changes.

## **B) $M_t$ is mostly unspanned by oil futures**

I project  $M_t$  on the first two principal components of log oil futures prices and label the residual  $UM_t$ :

$$M_t = \alpha + \gamma_{1,2}PC_t^{1,2} + UM_t$$

The  $R^2$  of the projections for  $GRO$  and  $INV$  are 6.4% and 27.5% . Projecting on the first five PCs the  $R^2$  are 14.5% and 30.0%, and projecting on all 12 individual futures maturities the  $R^2$  are 18.9% and 30.9%. Thus, much of the monthly variation in  $M_t$  – and particularly  $GRO$  – is unspanned by variation in oil futures prices. However,  $M_t$  might be measured with error or some subcomponents of  $M_t$  may be irrelevant to oil prices and risk premia. Thus the main question is not the projection  $R^2$  but whether  $M_t$  is relevant to returns and/or price forecasts, conditional on the information in futures prices.

## **C) $M_t$ forecasts returns over and above information in the futures curve**

Table 1 Panel A shows the results of forecasting returns to oil futures using information from the current futures curve and then adding the macro variables  $M_t$ . We see that  $M_t$  – particularly  $GRO$  – contributes additional forecasting power, conditional on the futures curve, for both returns to the second nearby contract (columns 1-3) and average returns to all traded futures that month, which is effectively the same as the return to the level portfolio (columns 4-6). In the same way, Panel B shows that  $M_t$  forecasts changes in the level, but not the slope. The coefficients on  $f^{1-12}$  in these regressions (not shown) have different signs on adjacent maturities, a clear sign of overfitting, yet  $M_t$  still contributes substantial forecasting power.

Table 1: Panel A shows the results of forecasting the log return to the second nearby oil futures contract ( $r_{t+1}^2$ ) and the average log return to all active oil futures with maturities from 2 to 12 months ( $\bar{r}_{t+1}$ ). Panel B shows the results of forecasting changes in the principal components of log prices. *GRO* is the Chicago Fed National Activity Index and *INV* is the log of U.S. crude oil stocks. The data are monthly from from 1/1986 to 6/2014. Newey-West standard errors with six lags are in parentheses.

Panel A: Forecasting Returns

$$r_{t+1} = \alpha + \beta_{GRO,INV} M_t + \beta_{\mathcal{P}} \mathcal{P}_t + \epsilon_{t+1}$$

	$r_{t+1}^2$			$\bar{r}_{t+1}$		
<i>GRO</i> <sub><i>t</i></sub>	0.0259** (0.0109)	0.0242** (0.0112)	0.0215* (0.0115)	0.0194** (0.0092)	0.0194** (0.0095)	0.0175* (0.0097)
<i>INV</i> <sub><i>t</i></sub>	0.040 (0.092)	0.037 (0.093)	0.035 (0.093)	0.018 (0.076)	0.009 (0.075)	0.007 (0.076)
Spanned Factors $\mathcal{P}_t$ :	<i>PC</i> <sup>1,2</sup>	<i>PC</i> <sup>1-5</sup>	<i>f</i> <sup>1-12</sup>	<i>PC</i> <sup>1,2</sup>	<i>PC</i> <sup>1-5</sup>	<i>f</i> <sup>1-12</sup>
<i>T</i>	341	341	341	341	341	341
Adj. $R^2(\mathcal{P}_t)$	0.4%	0.7%	4.6%	0.2%	0.1%	3.9%
Adj. $R^2(\mathcal{P}_t + M_t)$	3.3%	3.0%	6.2%	2.7%	2.3%	5.5%
<i>F</i> -ratio	6.1***	4.9***	3.9**	5.2***	4.7***	3.8**

Panel B: Forecasting PCs

$$\Delta PC_{t+1} = \alpha + \beta_{GRO,INV} M_t + \beta_{\mathcal{P}} \mathcal{P}_t + \epsilon_{t+1}$$

	$\Delta PC^1$ (Change in level)			$\Delta PC^2$ (Change in slope)		
<i>GRO</i> <sub><i>t</i></sub>	0.0653** (0.0315)	0.0659** (0.0326)	0.0599* (0.0331)	0.0089** (0.0042)	0.0079* (0.0045)	0.0067 (0.0046)
<i>INV</i> <sub><i>t</i></sub>	0.059 (0.259)	0.027 (0.257)	0.018 (0.260)	0.041 (0.053)	0.047 (0.048)	0.046 (0.045)
Spanned Factors $\mathcal{P}_t$ :	<i>PC</i> <sup>1,2</sup>	<i>PC</i> <sup>1-5</sup>	<i>f</i> <sup>1-12</sup>	<i>PC</i> <sup>1,2</sup>	<i>PC</i> <sup>1-5</sup>	<i>f</i> <sup>1-12</sup>
<i>T</i>	341	341	341	341	341	341
Adj. $R^2(\mathcal{P}_t)$	-0.4%	-0.5%	2.9%	6.5%	8.0%	10.3%
Adj. $R^2(\mathcal{P}_t + M_t)$	2.0%	1.6%	4.6%	7.5%	8.8%	10.8%
<i>F</i> -ratio	5.1***	4.7***	3.8**	1.3	1.1	0.8

The forecasting power of  $GRO$  is statistically significant. Adding  $GRO$  to the forecasting regression increases the adjusted  $R^2$  for first-nearby and average oil futures returns from 0.4% and 0.2% to 3.3% and 2.7% respectively: the adjusted  $R^2$  for the level of oil prices increases from -0.4% to 2.0%. The forecasting power of  $GRO$  is also economically significant. A one percent increase in  $GRO$ , about two standard deviations, forecasts a return to the level portfolio that is 194 basis points higher over the next month. However,  $GRO$  reverts quickly toward its mean, and oil futures are volatile – the monthly standard deviations of returns are 9.6% for  $r_t^2$  and 7.7% for  $\bar{r}_t$ . As a result, the corresponding implied Sharpe ratios are modest at 0.59 and 0.60 for the second nearby future and the level portfolio respectively. Thus, while real activity is a first-order determinant of oil futures returns, the level of implied predictability is not implausibly large given the volatility of oil futures.

Importantly, these results are not driven by the sample period or by the specific measure of real activity. Appendix A shows that the forecasting power of real activity for oil prices and returns is robust to using two alternative real activity measures, excluding the volatile period after 2007, including several measures of time-varying volatility, and using year-on-year changes to eliminate persistent regressors.

### 3 Model

Motivated by the stylized facts described in Section 2, this section develops a macro-finance model for commodity futures that admits unspanned macroeconomic state variables. Let  $X_t$  be a vector of  $N$  state variables that summarize the economy.  $X_t$  includes macroeconomic risk factors such as expected economic growth, and factors specific to the commodity such as hedging pressure, inventories, and expectations of supply and demand. The state vector

follows a Gaussian VAR,

$$X_{t+1} = K_{0X}^{\mathbb{P}} + K_{1X}^{\mathbb{P}}X_t + \Sigma_X \epsilon_{t+1}^{\mathbb{P}} \quad (1)$$

where  $\epsilon_{t+1}^{\mathbb{P}} \sim N(0, 1_N)$ . Risk premiums are “essentially affine” in the state variables (Duffee, 2002) so the stochastic discount factor is given by

$$\mathcal{M}_{t+1} = e^{(\Lambda_0 + \Lambda_1 X_t)' \epsilon_{t+1}} \quad (2)$$

This specification includes previous benchmark models such as Gibson and Schwartz (1990); Schwartz (1997); Casassus and Collin-Dufresne (2005). All of these previous models implicitly assume perfect spanning i.e. the state  $X_t$  is fully reflected in contemporaneous futures prices. As is well known for bond yields (Duffie and Kan, 1996), the spanning assumption implies that  $X_t$  can be replaced by an arbitrary set of linear combinations of log futures prices:

$$\mathcal{P}_t^N = W f_t$$

where  $W$  is *any* full rank  $N \times J$  matrix. Thus, the spanning assumption implies:

1. Futures prices are described up to idiosyncratic errors by the  $N$  factors  $\mathcal{P}_t^N$ .
2. The projection of  $X_t$  on  $\mathcal{P}_t^N$  has  $R^2$  of one.
3. Conditional on  $\mathcal{P}_t^N$ , no other information forecasts  $X_t$  or futures prices or returns.

I instead assume that a subspace of  $X_t$  is spanned, while its complement is unspanned but observed by the econometrician. Suppose that contemporaneous futures prices are determined by a set of linear combinations  $L_t = V f_t$  where  $V$  is a real valued  $N_L \times J$  matrix and  $N_L < N$ . That is, the spot price and its evolution under the risk neutral measure are given

by:

$$s_t = \delta_0 + \delta_1' L_t \quad (3)$$

$$L_{t+1} = K_{0L}^{\mathbb{Q}} + K_{1L}^{\mathbb{Q}} L_t + \Sigma_L \epsilon_{t+1}^{\mathbb{Q}} \quad (4)$$

where  $\epsilon_{t+1}^{\mathbb{Q}} \sim N(0, 1_{N_L})$  and  $\Sigma_L = V\Sigma_X$ .<sup>7</sup> By the same rationale as before we can replace  $L_t$  with  $N_L$  linear combinations of log prices,

$$\mathcal{P}_t^L = W_L f_t$$

where  $W_L$  is any full rank  $N_L \times J$  matrix, and transform the state space from  $X_t$  to  $(\mathcal{P}_t^L, UM_t)$  where  $UM_t$  are the unspanned components (projection residuals) of the macro factors  $M_t$ . In contrast to the perfect-spanning models, this model implies that:

1. Futures prices are described up to idiosyncratic errors by  $N_L < N$  factors.
2. The projection of  $X_t$  on  $\mathcal{P}_t^L$  has  $R^2$  less than one.
3. Conditional on  $\mathcal{P}_t^L$ , other information may forecast  $X_t$  or futures prices or returns.

Motivated by the variance decomposition in the previous section, I assume the number of spanned state variables  $N_L = 2$ . After estimating the model I rotate and translate so that the state variables correspond to the model implied spot price and cost of carry  $(s_t, c_t)$  and the macroeconomic series  $M_t$ .<sup>8</sup>

The model can then be described in just two equations:

- 1) the law of motion for the state variables:

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<sup>7</sup>This specification is in the class of “hidden-factor” models explored by [Diebold, Rudebusch and Aruoba \(2006\)](#); [Duffee \(2011\)](#); [Chernov and Mueller \(2012\)](#); [Joslin, Priebsch and Singleton \(2014\)](#) for bond yields.

<sup>8</sup>A detailed description of the parametrization and estimation are in the Internet Appendix.

$$\begin{bmatrix} s_{t+1} \\ c_{t+1} \\ M_{t+1} \end{bmatrix} = \begin{bmatrix} K_{0sc}^{\mathbb{P}} \\ K_{0M}^{\mathbb{P}} \end{bmatrix} + \begin{bmatrix} K_{sc,sc}^{\mathbb{P}} & K_{sc,M}^{\mathbb{P}} \\ K_{M,sc}^{\mathbb{P}} & K_{MM}^{\mathbb{P}} \end{bmatrix} \begin{bmatrix} s_t \\ c_t \\ M_t \end{bmatrix} + \Sigma \epsilon_{t+1}^{\mathbb{P}} \quad (5)$$

2) the dynamics of  $(s_t, c_t)$  under the risk neutral measure:

$$\begin{bmatrix} s_{t+1} \\ c_{t+1} \end{bmatrix} = K_0^{\mathbb{Q}} + K_1^{\mathbb{Q}} \begin{bmatrix} s_t \\ c_t \end{bmatrix} + \Sigma_{sc} \epsilon_{t+1}^{\mathbb{Q}} \quad (6)$$

The model is a canonical form, that is, any affine futures pricing model with two spanned state variables and  $N_M \geq 0$  macroeconomic variables can be written in the form above. Extending it to more than two spanned state variables is straightforward.

### 3.1 Time-Varying Volatility

The Gaussian model produces tractable affine pricing, but the volatility of commodity futures clearly varies over time ([Trolle and Schwartz \(2009\)](#)). Time varying volatility affects futures prices directly via the convexity term and might also affect price forecasts or expected returns. If these effects are present, in general they will be reflected in the reduced-form (pricing) factors because the model identifies the spanned state variables directly from futures prices in an agnostic way. Thus, any spanned effects of stochastic volatility on expected returns and price forecasts are compatible with the estimates and do not confound the findings.

Additional forecasting regressions also show that unspanned stochastic volatility does not explain the association between real activity and oil futures returns. [Appendix A](#) shows the results of forecasting regressions when I include measures of oil futures volatility. All three volatility measures are insignificant in forecasting oil futures returns and price changes,



and more importantly they do not alter the forecasting power of  $GRO$ . The fact that time-varying volatility does not subsume the forecasting power of  $GRO$  is also consistent with the finding of [Chiang, Hughen and Sagi \(2015\)](#) that exposure to crude oil volatility has a risk premium attached to it in equities, but not in oil futures.

## 4 Model Estimates

Table 2 presents the parameters of the maximum likelihood estimate of the model. The coefficient of  $\Delta s_{t+1}$  on  $GRO_t$  is positive and statistically significant at the 1% level, echoing the forecasting regressions in Table 1. Note that a higher value of  $GRO_t$  also forecasts a fall in the cost of carry  $\Delta c_{t+1}$ , whereas Table 1 Panel B shows that higher  $GRO_t$  weakly forecasts a higher value of the slope factor. The state variables  $(s_t, c_t)$  are not the same as the level and slope factors in futures prices, and hence a VAR using the principal components plus the macroeconomic factors would not deliver the same results.

Similar to the PCs, the model-implied spanned factors  $(s_t, c_t)$  do a good job of summarizing the term structure of oil futures prices.<sup>9</sup> The model fitted values for  $f_t$  explain 99.97% of variation in observed log futures prices and the residuals (pricing errors) explain 0.03%. The root mean squared pricing error (RMSE) of the model is 54 basis points, in line with benchmark futures pricing models.

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<sup>9</sup>This does not contradict the conclusions of e.g. [Schwartz \(1997\)](#) and [Casassus and Collin-Dufresne \(2005\)](#) that a three-factor model is necessary to summarize commodity futures prices. The three-factor models in those papers have two latent factors – spot price and convenience yield – and a spanned interest rate that is estimated separately. Interest rates are very slow moving compared to futures prices, so they contribute almost no explanatory power.

Table 2: Maximum likelihood (ML) estimate of the macro-finance model for Nymex crude oil futures using data from January 1986 to June 2014.  $s_t$ ,  $c_t$  are the spot price and annualized cost of carry respectively.  $GRO_t$  is the monthly Chicago Fed National Activity Index.  $INV_t$  is the log of the private U.S. crude oil inventory as reported by the EIA. The coefficients are over a monthly horizon, and the time series are de-meaned. ML standard errors are in parentheses.

	$K_0^{\mathbb{P}}$	$K_1^{\mathbb{P}}$			
		$s_t$	$c_t$	$GRO_t$	$INV_t$
$\Delta s_{t+1}$	0.009 (0.006)	-0.007 (0.008)	0.060* (0.031)	0.025*** (0.008)	0.045 (0.076)
$\Delta c_{t+1}$	-0.007 (0.005)	0.017** (0.008)	-0.119*** (0.029)	-0.015* (0.007)	-0.074 (0.071)
$\Delta GRO_{t+1}$	-0.002 (0.030)	-0.094** (0.044)	-0.022 (0.163)	-0.380*** (0.042)	0.028 (0.404)
$\Delta INV_{t+1}$	0.002 (0.002)	0.001 (0.002)	0.029*** (0.008)	-0.001 (0.002)	-0.089*** (0.020)

	$K_0^{\mathbb{Q}}$	$K_1^{\mathbb{Q}}$	
		$s_t$	$c_t$
$\Delta s_{t+1}$	-0.003 (0.007)	0.000 (0.003)	0.083*** (0.011)
$\Delta c_{t+1}$	-0.001 (0.011)	-0.004 (0.009)	-0.106*** (0.030)

	Shock Volatilities			
	$s$	$c$	$GRO$	$INV$
$s$	0.100			
$c$	-81%	0.056		
$GRO$	5%	-2%	0.527	
$INV$	-21%	25%	4%	0.026

## 4.1 Dynamics of the State Variables

The models of [Schwartz \(1997\)](#) and [Schwartz and Smith \(2000\)](#) impose the restriction that the spot price  $s_t$  is a random walk. Without that restriction and using data from 1990 to 2003, [Casassus and Collin-Dufresne \(2005\)](#) estimate that  $s_t$  is mean reverting with a half-life of two years, so the expected spot price of oil in ten years' time is effectively constant. The unrestricted estimate in [Table 2](#) which adds ten years of subsequent data is more consistent with [Schwartz \(1997\)](#). The coefficient of  $\Delta s_{t+1}$  on  $s_t$  is -0.007 and is statistically not distinct from zero.<sup>10</sup>

The cost of carry reverts to a slightly negative mean with a half-life of five months. Shocks to the spot price and the cost of carry are strongly negatively correlated ( $\rho = -81\%$ ), so a higher spot price is accompanied by a more downward sloping curve, but spot price shocks are essentially permanent while the cost of carry shock decays within a few years. As a result, about half of a typical move in the oil spot price disappears after two to three years, while the other half is expected to persist effectively forever.

[Figure 3 Panel A](#) plots the components of  $GRO$  that are spanned and unspanned by  $(s_t, c_t)$ . We see that effectively all of the monthly and yearly variation in  $GRO$  appears in the unspanned component. [Figure 3 Panel B](#) plots the spanned and unspanned components of log oil inventories  $INV$ . Compared to  $GRO$ , much more of the monthly and yearly variation in  $INV$  is spanned by log futures prices. The spanned component of inventory loads exclusively and strongly on the cost of carry  $c_t$ .

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<sup>10</sup>The estimates all use nominal futures prices. Inflation was slow moving over the period from 1986 to 2014 relative to movements in oil prices, so it has little effect on the dynamics of  $s_t$ ; using futures prices deflated by the CPI or PPI does not change any of the estimates in the paper.

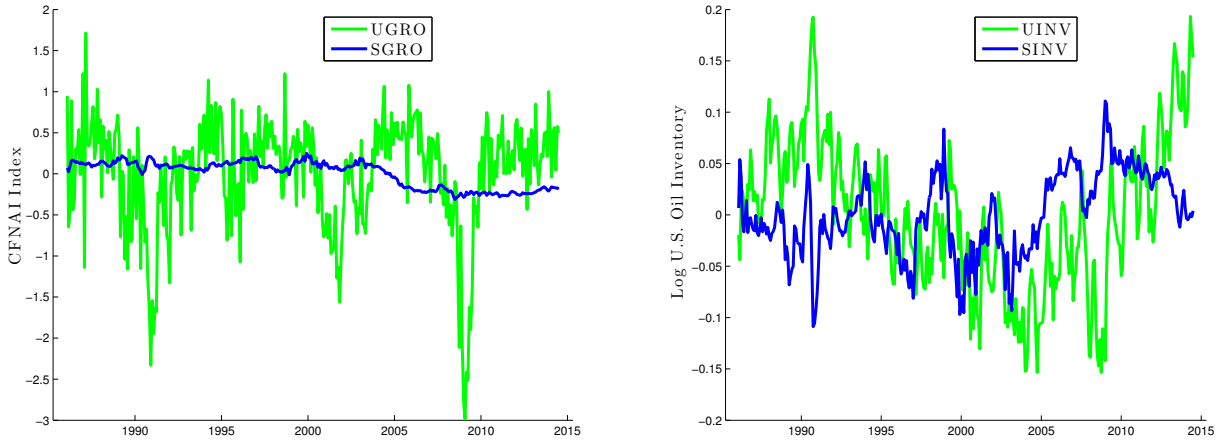


Figure 3: Panel A plots the components of the monthly Chicago Fed National Activity Index  $GRO$  that are spanned ( $SGRO$ ) and unspanned ( $UGRO$ ) by log oil futures prices. Panel B plots the components of monthly log U.S. oil inventories  $INV$  that are spanned ( $SINV$ ) and unspanned ( $UINV$ ) by log oil futures prices.

#### 4.1.1 Oil Prices and Real Activity

A one percent shock to real activity forecasts a 2.5% higher spot price of oil and a 1.5% lower cost of carry. On net, the effects of real activity on oil prices are expected by the market to be persistent – higher real activity raises both the short run and the expected long run price of oil.

Conversely, a higher spot price of oil forecasts lower real activity. A higher cost of carry – higher expected prices in future – forecasts slightly higher real activity, but  $c_t$  naturally forecasts a higher spot price as well. The impulse response functions in Section 4.1.3 make clear that the net effect of  $c_t$  on  $GRO$  is negative. As a result, a shock to the spot price of oil that the market expects to persist has a more negative effect on growth than a shock that is expected to be transitory.

In sum, there is a negative feedback relationship between the spot price of oil and real economic activity. A positive shock forecasts persistent higher oil prices, while a positive oil

price shock forecasts lower real activity, and the effect is stronger for oil price shocks that the market expects to persist.

#### 4.1.2 Oil Prices and Inventories

Shocks to log inventories are negatively correlated with the spot price and positively correlated with the cost of carry. Both of these observations are consistent with the Theory of Storage – higher inventories signal that the market is moving up the supply-of-storage curve. The correlation between shocks to inventory and the cost of carry (27%) is relatively modest; in the frictionless storage model of [Working \(1949\)](#) and others,  $INV_t$  and  $c_t$  are collinear. A higher cost of carry strongly predicts higher inventories the next month. The forecasting power of  $c_t$  for inventories suggests adjustment costs in production and storage (e.g. [Carlson, Khokher and Titman \(2007\)](#)): the futures curve adjusts to new information first and inventories respond with a lag.

Looking down the last column of the transition matrix, unspanned crude oil inventory does not forecast any of the other variables. In particular, periods of higher inventory do not have much effect on the forecast of either the spot price or the cost of carry. This finding is consistent with the fundamental drivers of oil inventory such as precautionary storage and expected physical supply and demand being fully spanned by oil futures prices.

#### 4.1.3 Impulse Response Functions

Figure 4 plots the impulse response functions (IRFs) to shocks to oil prices and economic activity. The ordering of the variables for the impulse response functions is  $(GRO, s_t, c_t, INV)$ .  $GRO$  is first because innovations in the unspanned component, which dominates the variation in  $GRO$ , can be thought of as exogenous to contemporaneous oil prices and inventories.

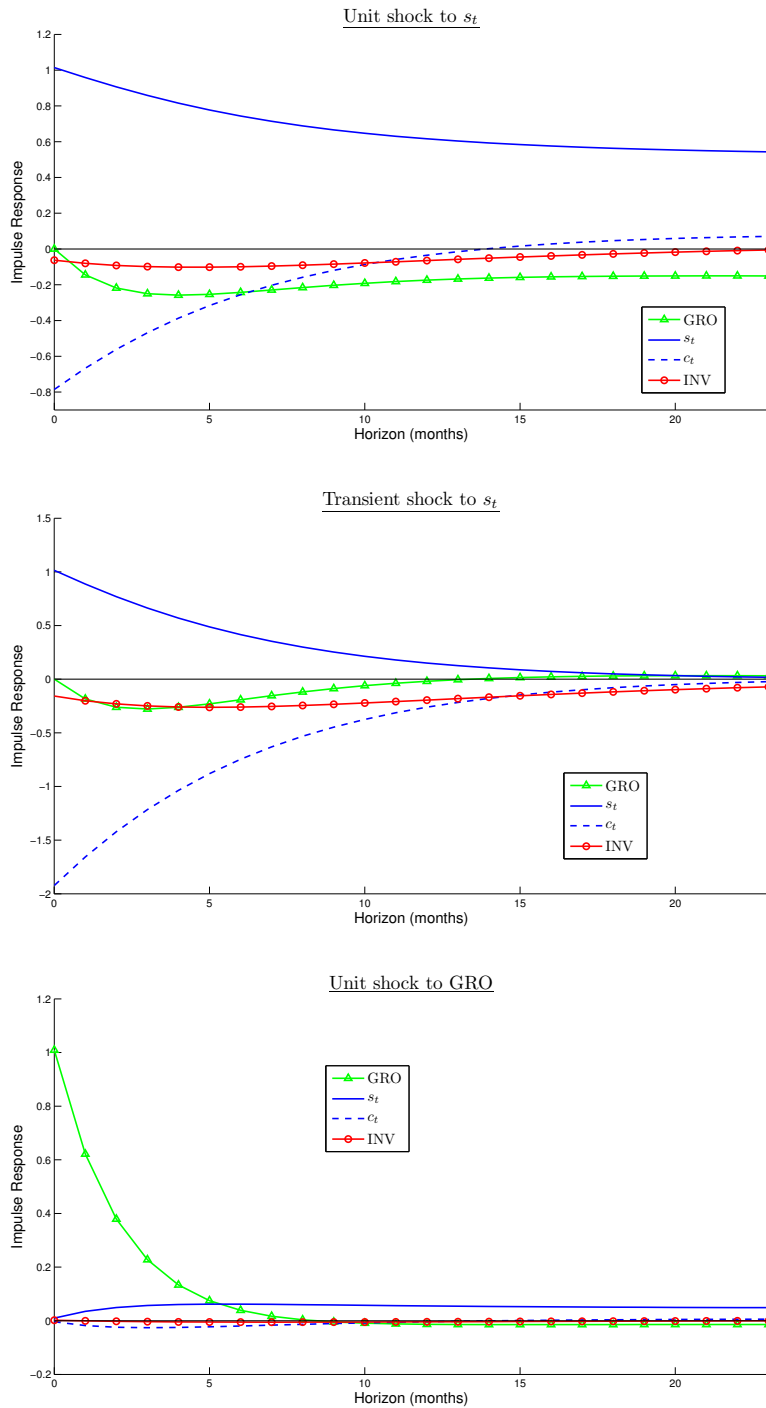


Figure 4: Panel A shows the impulse response functions (IRFs) of the four state variables to a unit shock to the log spot price of oil  $s_t$ . Panel B shows the IRFs for a transient shock for which the spot price of oil fully reverts to the baseline. Panel C shows the IRFs for a unit shock to economic growth,  $GRO$ . The order of the variables is  $(GRO, s_t, c_t, INV)$ .

We analyze  $s_t$  and  $c_t$  simultaneously so their relative ordering is not important. Finally, it is intuitive and also supported by the estimates and regressions that the oil futures curve adjusts to new information faster than physical inventory does.

Panel A plots the response to a unit shock to the log spot price, which is correlated with a negative shock to the cost of carry and a more downward-sloping curve. A unit shock to  $s_t$  means a doubling of the spot price of oil. About half of the increase decays within two years, while the other half is effectively permanent, and forecasts an economic activity index that is 0.2% lower effectively forever. This effect is material: the index averaged -1.66% in 2009 during the depths of the financial crisis, while it averaged 0.02% in 2006. The higher spot price and lower cost of carry also produce a fall in inventories.

Panel B plots the response to a joint shock to  $s_t$  and  $c_t$  such that the spot price is expected to fully revert to the pre-shock baseline. The response of economic activity is transient as well, and in fact  $GRO$  recovers to the baseline faster than  $s_t$  does. Comparing to Panel A, which only differs in the size of the shock to  $c_t$ , makes clear that the net effect of  $c_t$  on expected growth is negative. Note that the fact that the forecast of the long-run spot price is unchanged in Panel B does not mean that long maturity futures prices will be unchanged – the two are equivalent only in the case that oil risk premiums are non time varying. Thus, a VAR that includes a long-maturity futures price or spread will not in general recover the correct dynamics of the state variables.

Panel C plots the response to a shock to economic activity. The index mean reverts rapidly and the shock decays back to the baseline within a year. However, a transient shock to  $GRO$  produces a near permanently higher spot price of oil – perhaps because oil is a nonrenewable resource. The magnitude of the effect is large: a one-period shock to economic activity of one percent produces a spot price of oil that is 5.1% higher than the baseline, ten

Table 3: Maximum likelihood (ML) estimates of the parameters governing risk premiums in the macro-finance model for U.S. crude oil futures.  $s_t$ ,  $c_t$  are the spot price and annualized cost of carry respectively.  $GRO$  and  $INV$  are the Chicago Fed National Activity Index and log U.S. crude oil inventory respectively. The coefficients are standardized to reflect a one standard deviation change in each variable over a monthly horizon, and the time series are de-meaned. ML standard errors are in parentheses.

$$\begin{bmatrix} \Lambda^s \\ \Lambda^c \end{bmatrix}_t = \Lambda_0 + \Lambda_1 \begin{bmatrix} s_t & c_t & M_t \end{bmatrix}'$$

	$\Lambda_0$	$\Lambda_1$			
		$s_t$	$c_t$	$GRO_t$	$INV_t$
$\Lambda^s$	0.011 (0.013)	-0.001 (0.001)	-0.001 (0.002)	0.013*** (0.004)	0.001 (0.002)
$\Lambda^c$	-0.006 (0.016)	0.003 (0.002)	-0.001 (0.003)	-0.008* (0.004)	-0.002 (0.002)

years later.

## 4.2 Risk Premiums

Table 3 displays the estimates of the parameters governing time-varying risk premiums. The unconditional spot risk premium is positive, while the unconditional cost-of-carry risk premium is negative.<sup>11</sup> Two entries in the time-varying loadings of risk premiums  $\Lambda_1$  are statistically significant: higher real activity  $GRO$  is associated with a higher spot risk premium and a lower cost-of-carry risk premium in oil. As in the forecasting regressions, this finding is inconsistent with the risk spanning assumption that is implicit in previous pricing models.

<sup>11</sup>Szymanowska et al. (2013) decompose futures returns into a spot premium and a term premium based on different trading and rolling strategies. The Internet Appendix describes the relation between their decomposition and the risk premiums in the model. Briefly, their spot premium equals the risk premium attached to the spot price plus a small convexity term, while their term premium equals the risk premium attached to the cost of carry minus the conditional expected cost of carry.



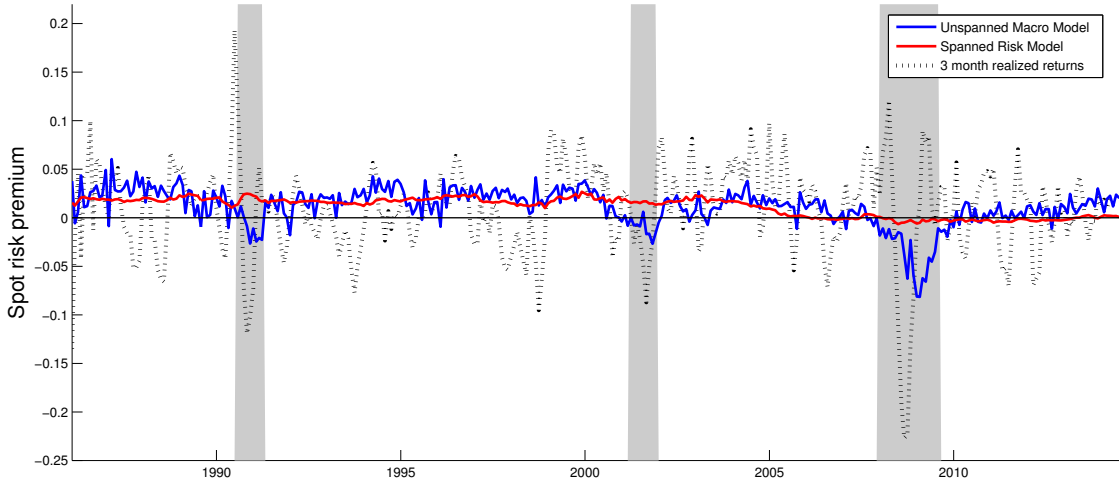


Figure 5: The figure compares the spot risk premium in oil futures from the model with unspanned macro factors versus the nested model that enforces spanning. Also plotted is the average return across all active oil futures over the following three months. NBER recessions are shaded in grey.

The effect of economic activity on the estimated spot risk premium in oil futures is material. Figure 5 plots the implied spot premiums for the macro-finance model and the two-factor nested model that enforces spanning, as well as average realized returns for oil futures in the sample over the following three months. The model predictions differ most noticeably during the recessions of 1990-1991, 2001-2002 and 2008-2009: slumps in real activity forecast falling oil prices. The unspanned procyclical component dominates the variation in the spot risk premium; the standard deviation of changes in  $\Lambda_t^s$  in the unspanned macro model is 1.47% per month compared to 0.17% per month in the spanned-risk model, an increase of nearly tenfold.

The macro-finance model does significantly better at explaining both returns to the level portfolio, as Figure 5 suggests, and the full panel of futures returns. Taking the panel of log

futures returns and comparing them with their predicted values:

$$r_{t+1}^j = f_{t+1}^{j-1} - f_t^j, \quad j = 2, \dots, J, \quad t = 0, \dots, T - 1$$

$$\lambda_{t+1}^j = B_j \Lambda_t$$

$$resid_{t+1}^j = r_{t+1}^j - \lambda_{t+1}^j$$

the adjusted  $R^2$  of the macro-finance model is 4.3% compared to 1.3% for the nested spanned-risk model. Thus, after adjusting for the added degrees of freedom, adding the unspanned macro factors more than triples the fraction of oil futures returns that the model is able to explain.

The forecasting power for returns is attached to the unspanned component of real activity because it is not reflected in the futures curve at the time. Per the estimates in Table 2, shocks to real activity and the cost of carry are negatively correlated. A fall in  $GRO$  is correlated with a fall in the spot price  $s_t$  but a slight rise in  $c_t$  and the slope of the futures curve. In other words, when real activity falls, long maturity oil futures fail to forecast the subsequent fall in the spot price.

### 4.3 Oil Futures as a Consumption Hedge

A potential motivation for the procyclical expected returns in oil futures is as follows. Suppose a representative agent consumes oil  $O_t$  and a general consumption good  $C_t$ , with

$$V_t = E \left[ \sum_{t=0}^{\infty} \beta^t \frac{A_t^{1-\gamma}}{1-\gamma} \right]$$

$$A(C_t, O_t) = [C_t^{1-\rho} + \omega O_t^{1-\rho}]^{\frac{1}{1-\rho}}$$

I assume that  $\gamma > \rho > 1$ , that is, oil and the general consumption good are complements and investors' risk aversion over total consumption is stronger than the elasticity between the two goods. Normalize the price of the consumption good to 1 and denote by  $P_t^O$  the price of a barrel of oil. The intratemporal equilibrium is

$$P_t^O = \omega \left( \frac{C_t}{O_t} \right)^\rho$$

and the stochastic discount factor is<sup>12</sup>

$$\mathcal{M}_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{1 + \omega O_{t+1}^{1-\rho} / C_{t+1}^{1-\rho}}{1 + \omega O_t^{1-\rho} / C_t^{1-\rho}} \right)^{\frac{\rho-\gamma}{1-\rho}}$$

Log linearizing,

$$m_t \approx K - \gamma - \omega(\gamma - \rho)\Delta c_{t+1} - \omega(\gamma - \rho)\Delta o_{t+1}$$

where  $K$  is a constant. The expected log return on any asset is

$$E[r_{t+1}] = r_t^f + (\gamma - \omega(\gamma - \rho))cov(r_{t+1}, \Delta c_{t+1}) + \omega(\gamma - \rho)cov(r_{t+1}, \Delta o_{t+1})$$

where  $r_t^f$  is the log risk free rate. Thus, assets pay a consumption risk premium for exposure to  $c_t$  shocks and an oil risk premium for exposure to  $o_t$  shocks. Note that this prediction does not assume anything about the dynamics of  $c_t$  or  $o_t$  – consumption of oil and the general good may be exogenous or endogenous.

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<sup>12</sup>This derivation follows, e.g., [Yogo \(2006\)](#).

Oil futures have a loading that is close to unity on the spot price of oil and hence a loading of  $-\rho$  on  $o_t$ . Thus, consumers pay a hedge premium to be long oil futures because it insures them against shocks to the flow of oil. If risk aversion  $\gamma$  is time-varying and higher in recessions, as is well established across asset markets (Cochrane, 2011), then expected returns to oil futures will be procyclical because the hedge premium is higher in recessions.<sup>13</sup>

Both regressions and the model estimate indicate that real activity is not reflected in contemporaneous oil futures prices.<sup>14</sup> This corresponds to real activity having offsetting effects on oil risk premiums and the spot price forecast. We observe in the data that higher real activity is followed by higher spot prices of oil but that oil futures do not slope upward to reflect that forecast. A positive growth shock lowers the hedge premium, which *lowers* the futures price. At the same time, it forecasts higher demand and a higher spot price of oil, which *raises* the futures price, consistent with the net effect being unspanned in the contemporaneous futures curve.

#### 4.4 Hedging Pressure: Producers versus Consumers

The hypothesis outlined above suggests that real activity drives oil risk premia via demand for hedging oil consumption. This explanation is in contrast with the “hedging pressure” theory that originated with Keynes (1930), which says that risk premia in commodity futures markets are due to hedging demand from commodity producers.

The track record of hedging pressure as an explanation of risk premia in commodity futures markets is poor. Most studies find that hedging pressure does not forecast returns or prices in most futures markets (e.g. Bessembinder (1992); Gorton, Hayashi and Rouwenhorst

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<sup>13</sup>In contemporaneous research Ready (2015) and Baker, Steven D. and Routledge (2015) solve models that generate an endogenously time-varying hedge premium to oil futures.

<sup>14</sup> The existence of macro variables that are unspanned by bond yields is an active question in the term structure literature (Duffee, 2011; Chernov and Mueller, 2012; Joslin, Priebsch and Singleton, 2014).

Table 4: Maximum likelihood (ML) estimate of the transition matrix for the model in which  $M_t = [GRO, HP]$  where  $GRO$  and  $HP$  are the monthly Chicago Fed National Activity Index and the hedging pressure calculated from the CFTC’s Commitment of Traders reports. The coefficients are over a monthly horizon, and the time series are de-meanned. ML standard errors are in parentheses.

	$K_0^{\mathbb{P}}$	$K_1^{\mathbb{P}}$			
		$s_t$	$c_t$	$GRO_t$	$HP_t$
$\Delta s_{t+1}$	0.007 (0.006)	-0.012 (0.009)	0.075*** (0.028)	0.023*** (0.008)	-0.083 (0.069)
$\Delta c_{t+1}$	-0.006 (0.005)	0.022** (0.009)	-0.139*** (0.026)	-0.013 (0.008)	0.088 (0.065)
$\Delta GRO_{t+1}$	-0.018 (0.030)	-0.148*** (0.050)	0.040 (0.146)	-0.395*** (0.043)	-0.776** (0.364)
$\Delta HP_{t+1}$	-0.010 (0.003)	-0.012** (0.005)	-0.037** (0.016)	-0.011** (0.005)	-0.257*** (0.039)

(2013)). I reach the same conclusion. Table 4 shows the estimated parameters that govern oil risk premia in the model using  $M_t = [GRO_t, HP_t]$  where  $HP$  is the hedging pressure in oil futures computed from the CFTC’s Commitment of Traders Report.<sup>15</sup> We see that hedging pressure does not drive oil risk premia, nor does it subsume the strong relationship of real activity with the oil risk premium. On the other hand, we see that higher current oil prices  $s_t$  or expected future oil prices via  $c_t$  both forecast less hedging pressure i.e. fewer commercial participants going long. This conclusion based on the time series of oil futures returns across the full term structure of oil futures is consistent with [Gorton, Hayashi and Rouwenhorst \(2013\)](#)’s conclusion using portfolio sorts across the cross-section of commodity futures.

<sup>15</sup>Hedging pressure equals the net position among commercial participants divided by the total open interest, as of the most recent Commitment of Traders report.

## 4.5 Positive vs Negative Growth Regimes

Looking at Figure 5, the effect of real activity on oil prices appears to be concentrated in economic downturns. To investigate this possibility I split  $GRO$  into two components:  $GRO^+$  equals  $GRO$  in months when the lagged three month average of  $GRO$  is positive and equals zero otherwise, and  $GRO^-$  equals  $GRO$  in months when the lagged three month average of  $GRO$  is negative and equals zero otherwise.<sup>16</sup> This split lets the coefficients differ when real activity is in a positive-growth regime versus a negative-growth regime.

Table 5 presents the estimated feedback matrix  $K_1^{\mathbb{P}}$ . Contrary to our impression from Figure 5, the effect of real activity on the oil prices seems to be symmetrical in good times versus bad – the point estimates of the coefficients of  $\Delta s_{t+1}$  on  $GRO^-$  and  $GRO^+$  are precisely equal. On the other hand, the estimate suggests an asymmetry on the 'supply' side. The forecast of real activity is impaired by oil price shocks in positive growth regimes: the coefficient of  $\Delta GRO_{t+1}^+$  on  $s_t$  is -0.082 and is statistically significant at the 1% level. By comparison the coefficient of  $\Delta GRO_{t+1}^-$  on  $s_t$  i.e. in negative growth regimes is less than half as large and is not statistically significant. Thus, while the effect of shocks to real activity on the oil price forecast is relatively symmetric in good times and bad, the negative effect of oil price shocks on the forecast of real activity is stronger in times when growth is positive.

## 4.6 Subcomponents of Real Activity

The Chicago Fed divides the 85 constituent time series of the CFNAI into four categories which form subcomponents of the real activity index. The subcomponents are Industrial Production and Income (PI); Employment and Hours (EUH); Personal Consumption and

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<sup>16</sup>As the unspanned components of the macroeconomic series do not enter into contemporaneous prices, they need not be Gaussian.

Table 5: Maximum likelihood (ML) estimate of the transition matrix for the model in which  $GRO^+$  and  $GRO^-$  are the monthly Chicago Fed National Activity Index in months when the lagged three month average of  $GRO$  is positive and negative respectively. The coefficients are over a monthly horizon, and the time series are de-meanned. ML standard errors are in parentheses.

	$K_0^{\mathbb{P}}$	$K_1^{\mathbb{P}}$			
		$s_t$	$c_t$	$GRO_t^+$	$GRO_t^-$
$\Delta s_{t+1}$	0.009 (0.007)	-0.006 (0.008)	0.069** (0.027)	0.025 (0.016)	0.025** (0.010)
$\Delta c_{t+1}$	-0.009 (0.006)	0.016** (0.008)	-0.133*** (0.026)	-0.009 (0.015)	-0.018* (0.010)
$\Delta GRO_{t+1}^+$	0.147 (0.022)	-0.082*** (0.028)	-0.048 (0.092)	-0.789*** (0.052)	0.076** (0.034)
$\Delta GRO_{t+1}^-$	-0.052 (0.025)	-0.038 (0.031)	0.052 (0.102)	0.036 (0.058)	-0.281*** (0.038)

Housing (CH); and Sales Orders and Inventories (SOI). When I estimate the model with the four subcomponents  $M_t = [PI_t, EH_t, PCH_t, SI_t]$  I find that employment  $EUH$  and sales and inventories  $SOI$  do not interact significantly with oil prices, but that industrial production  $PI$  and personal consumption  $PCH$  do.

Table 6 presents the results of estimating the model using those two subcomponents of the real activity index,  $M_t = [PI_t, PCH_t]$ . We see that the “demand” channel in which shocks to real activity affect the oil price forecast appears to be driven through industrial production. The coefficient of  $\Delta s_{t+1}$  on  $PI_t$  is positive (0.141) and statistically significant while the coefficient of  $\Delta s_{t+1}$  on  $PCH_t$  is much smaller and is not statistically significant. Also, shocks to  $PI$  are contemporaneously correlated with a higher spot price of oil  $s_t$ . By contrast the “supply” channel in which shocks to the oil price affect real activity appears to be driven by personal consumption. The coefficient of  $\Delta PCH_{t+1}$  on  $s_t$  is negative and

Table 6: Maximum likelihood (ML) estimate of the dynamics for the model in which  $M_t = [PI_t, PCH_t]$  are the subcomponents of the Chicago Fed National Activity Index that measure industrial production and income ( $PI$ ) and personal consumption and housing ( $PCH$ ) respectively. The coefficients are over a monthly horizon, and the time series are de-meaned. ML standard errors are in parentheses.

	$K_0^{\mathbb{P}}$	$K_1^{\mathbb{P}}$			
		$s_t$	$c_t$	$PI_t$	$PCH_t$
$\Delta s_{t+1}$	0.009 (0.006)	-0.007 (0.009)	0.069** (0.029)	0.141** (0.053)	0.023 (0.039)
$\Delta c_{t+1}$	-0.008 (0.005)	0.015 (0.009)	-0.135*** (0.027)	-0.085 (0.050)	-0.021 (0.037)
$\Delta PI_{t+1}$	0.001 (0.006)	-0.001 (0.009)	0.024 (0.028)	-0.728*** (0.053)	0.070* (0.039)
$\Delta PCH_{t+1}$	-0.001 (0.003)	-0.013** (0.006)	-0.016 (0.017)	-0.047 (0.032)	-0.090*** (0.023)

	Shock Volatilities			
	$s$	$c$	$PI$	$PCH$
$s$	0.101			
$c$	-81%	0.056		
$PI$	15%	-10%	0.098	
$PCH$	3%	-2%	25%	0.057

statistically significant, and the coefficient of  $\Delta PCH_{t+1}$  on  $c_t$  (expected future oil prices) is also negative, while the coefficient of  $\Delta PI_{t+1}$  on  $s_t$  is much smaller and is not statistically significant.

This decomposition suggests another asymmetry in the dynamic between oil prices and the real economy. On one hand, shocks to industrial productivity are associated with higher oil prices both contemporaneously and with a lag, while shocks to consumer spending have little or no effect. On the other hand, oil shocks affect real activity through falling consumption spending which reacts with a lag to a higher spot price.



## 5 Real Options

Firms' capacity to adjust their investment or production ex post make up a substantial part of firm value, and evaluating and exercising those adjustments is a primary role of firm management (Pindyck (1988); Berk, Green and Naik (2004); Carlson, Fisher and Giammarino (2004)). Previous studies such as Brennan and Schwartz (1985) and Casassus and Collin-Dufresne (2005) have explored what commodity derivatives markets can tell us about the value of real options. These studies, which use spanned-risk models, make the implicit assumption that all relevant factors are spanned by commodity futures. In Trolle and Schwartz (2009) and Chiang, Hughen and Sagi (2015), a latent volatility factor is unspanned in oil futures but spanned by oil options.

By contrast, if a macro factor  $M_t$  is unspanned in the sense of this paper then it does not affect the price of *any* derivatives on the underlying commodity. Unspanned factors of this type are still relevant to real options, however, if the option's payoff depends on  $M_t$ . For example, an oil well is often modeled as the right to pump oil out of the ground at a fixed cost per barrel, equivalent to a purely financial option. But for a real oil well the costs of extraction are uncertain. Moel and Tufano (2002) find that for gold mines, changing extraction costs over time are a significant predictor of mine openings and closings after controlling for factors estimated from both futures and options on gold. More generally, commodity prices are only one element of the firm's decision process. For example, in an airline's decision to purchase more fuel efficient planes, the cost savings will vary with oil prices while revenues will vary with aggregate economic activity. Pindyck (1993) makes this argument and points out that relevant macroeconomic risk factors will affect real option valuation<sup>17</sup>.

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<sup>17</sup> "... this effect [of uncertainty on option value and exercise] is magnified when fluctuations in construction

I model an oil well as a ten year strip of European options on an oil field that produces 1000 barrels of oil per month when open. The oil is extracted at lifting cost  $L_t$  and sold at the spot price  $S_t$ . Thus, it is open in any month when  $S_t > L_t$ . To examine differences in valuation between models I follow a similar approach to that of [Carlson, Khokher and Titman \(2007\)](#). I simulate data for the state variables  $(s, c, GRO)$  and the log lifting cost  $l_t$  which is a function of all three state variables. [Appendix B](#) describes the setting in detail. I then estimate the macro-finance model in this paper as well as benchmark spanned-risk models on the simulated data, and compute the implied value of the well over a range of current lifting costs  $L_0$ .

[Figure 6](#) plots the real option value of wells under the different models across a range of current lifting costs. The lower two lines represent values for the benchmark spanned-risk models in which all relevant risks are assumed to be spanned by oil futures. This means that  $l_t$  must be a linear combination of  $s_t$  and  $c_t$  plus an error term ([Joslin, Le and Singleton \(2013\)](#)). The figure shows that when  $l_t$  is assumed to be spanned, whether the error term is modelled as an i.i.d. or AR(1) process is irrelevant to option valuation.

The upper two lines plot real option values from models that incorporate unspanned macro risk. The values are considerably higher; that is, the spanned-risk models miss a large component of real option value. Note that the volatility of  $l_t$  in the spanned-risk model is the same as it is in the unspanned-risk models: the difference is that  $l_t$ 's covariance with  $GRO$  adds a *persistent* unspanned component to lifting costs, which is ignored by the spanned-risk model. This has a large effect on option valuation: Adding the unpriced ( $\lambda = 0$ ) unspanned macro risk raises the real option value by 35% for an 'in the money' well with a current

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*costs are correlated with the economy, or, in the context of the Capital Asset Pricing Model, when the 'beta' of cost is high... [A] higher beta raises the discount rate applied to expected future costs, which raises the value of the investment opportunity as well as the benefit from waiting rather than investing now."*

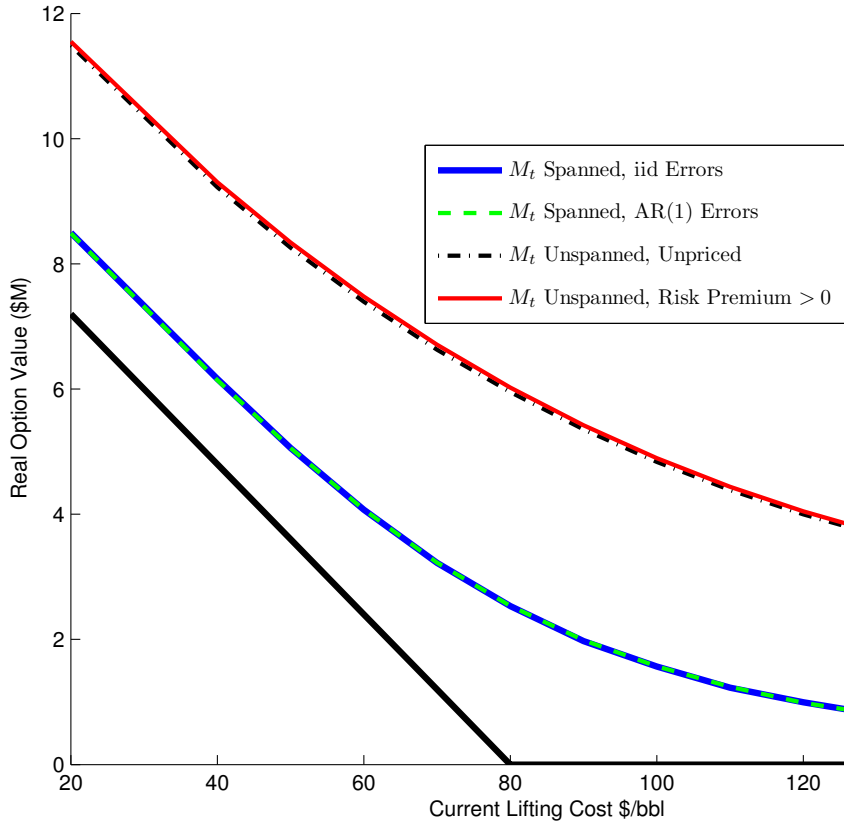


Figure 6: Real options valuation with unspanned macro risk. An oil well is modelled as a strip of European options that are exercised when the stochastic log extraction cost  $l_t$  is less than the log spot price  $s_t$ .  $l_t$  covaries with  $s_t$  and the unspanned macro risk  $GRO_t$ . The current spot price of oil is \$80, and the x-axis indexes the current lifting cost  $L_0$  of different oil wells.

lifting cost of \$20 per barrel and 405% for an 'out of the money' well with a current lifting cost of \$150 per barrel.

The risk premium effect is that the option value is higher when  $GRO$ , and hence  $L_t$ , carries a positive risk premium ( $\lambda > 0$ ). This effect on valuation is present but smaller, increasing the value by 1% for the 'in the money' well with  $L_0 = 20$  and by 1.27% for the 'out of the money' well with  $L_0 = 150$ .

## 6 Conclusion

This paper investigates the interaction of oil futures markets with the real economy. I develop a macro-finance futures pricing model that admits unspanned macroeconomic variables. The model generalizes and extends benchmark futures pricing models, and connects them with macroeconomic studies that use vector autoregressions (VARs).

Both forecasting regressions and model estimates reveal a novel empirical fact: real activity forecasts oil prices and the effect is unspanned in contemporaneous oil futures. The estimated oil risk premium in the model is strongly procyclical and nine times more volatile than the nested model that assumes complete spanning, which suggests that spanned-risk models may miss the majority of variation in futures risk premia. This pattern in the oil risk premium is consistent with countercyclical hedging demand by oil consumers.

By construction, unspanned macro factors do not affect the price of any commodity derivatives. However, when the payoff of a real option depends on macroeconomic factors beyond the commodity price then unspanned macro factors can have a large effect on real option value and exercise. In a calibrated example I show that both the dynamics and the risk premiums of unspanned macro risks raise the values of a hypothetical real option

significantly relative to a benchmark spanned-risk model.

The model estimates reveal rich dynamics of oil prices with the economy. Higher oil prices forecast lower real activity, especially when the price increase is forecast by the market to be persistent, and especially when the economy is in an expansion period. Higher real activity forecasts a higher oil price, and although real activity shocks are transient the resulting higher oil price is highly persistent, which may be related to the fact that oil is a nonrenewable resource. The channel from real activity to oil prices flows through industrial production, while the channel from oil prices to real activity flows through consumer spending.

To my knowledge these are the first estimates of a macro-finance style model for commodity futures which incorporates explicit macroeconomic factors. The estimates presented in this paper are all maximally flexible, meaning that the model imposes no overidentifying restrictions on the dynamics or risk prices of the state variables. Thus, the estimates are best seen as summarizing the data in a consistent and parsimonious way. A natural direction for further research will be to impose well-motivated restrictions on the dynamics of the state variables as in the VAR literature ([Barsky and Kilian, 2004](#); [Kilian, 2009](#)) and on risk prices as in the futures pricing literature ([Schwartz, 1997](#); [Casassus, Liu and Tang, 2013](#)), and further explore their interactions.

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## A Robustness Checks

### A.1 Alternative Measures of Real Activity

The predictability I find using the Chicago Fed National Activity Index also holds using other forward-looking measures of real activity. In this section I show that the same results obtain when I use the Aruba-Diebold-Scotti (ADS)<sup>18</sup> index (Aruoba, Diebold and Scotti, 2009) and the Conference Board’s Leading Economic Index (LEI)<sup>19</sup> in place of the CFNAI.

The LEI is a weighted forward-looking index of real activity like the CFNAI, but uses different weights and macroeconomic time series. The ADS index is a real-time forward-looking index of real activity that is extracted by filtering from a third set of macroeconomic time series. The time series are similar because all three are intended as forward-looking measures of real activity, but they are not identical: the correlation between the ADS index and the CFNAI is 83.8% in levels and 58.7% in changes while the correlation between the LEI and the CFNAI is 8.6% in levels and 25.3% in changes.

Table 7 shows the results of the return forecasting regressions using the ADS index (Panel A) and the LEI (Panel B). We see that both alternative indices positively forecast oil futures returns, over and above the information in the futures curve.

Table 8 shows the feedback matrix  $K_1^{\mathbb{P}}$  implied by estimating the affine model using the ADS index and the LEI in place of the CFNAI. As the CFNAI does, both the ADS index and the LEI forecast a higher spot price of oil (top right) and the spot price of oil negatively forecasts a lower value of the index (bottom left). Thus, the main conclusions are the same using alternative measures of real activity.

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<sup>18</sup><https://www.philadelphiafed.org/research-and-data/real-time-center/business-conditions-index/>

<sup>19</sup><https://www.conference-board.org/data/bcicountry.cfm?cid=1>

Table 7: Panel A shows the results of forecasting returns to oil futures.  $r_{t+1}^2$  is the log return to the second nearby oil futures contract and  $\bar{r}_{t+1}$  is the average log return to all active futures contracts with maturities from 2 to 12 months. The forecasting variables are 1) three sets of 'reduced-form' state variables  $\mathcal{P}_t$  based on oil futures prices and 2) the Aruoba-Diebold-Scotti index  $ADS_t$  (Panel A) or the Leading Economic Index ( $LEI_t$ ) plus log oil inventory  $INV_t$ . The data are monthly from from 1/1986 to 6/2014. Newey-West standard errors with lags are in parentheses.

Panel A: Aruoba-Diebold-Scotti (ADS) Index

$$r_{t+1} = \alpha + \beta_{ADS,INV}M_t + \beta_{\mathcal{P}}\mathcal{P}_t + \epsilon_{t+1}$$

	$r_{t+1}^2$			$\bar{r}_{t+1}$		
$ADS_t$	0.0317*** (0.0109)	0.0295*** (0.0113)	0.0293*** (0.0116)	0.0247** (0.0098)	0.0239** (0.0101)	0.0238** (0.0104)
$INV_t$	-0.002 (0.002)	-0.002 (0.002)	0.782 (0.644)	0.000 (0.001)	0.000 (0.001)	0.252 (0.573)
Spanned Factors $\mathcal{P}_t$ :	$PC^{1,2}$	$PC^{1-5}$	$f^{1-12}$	$PC^{1,2}$	$PC^{1-5}$	$f^{1-12}$
$T$	341	341	341	341	341	341
Adj. $R^2(\mathcal{P}_t)$	0.4%	0.7%	4.6%	0.2%	0.1%	3.9%
Adj. $R^2(\mathcal{P}_t + M_t)$	3.9%	3.6%	7.4%	3.5%	3.1%	6.7%
$F$ -ratio	6.6***	5.5***	5.4***	6.3***	5.6***	5.5***

Panel B: Conference Board Leading Economic Index (LEI)

$$r_{t+1} = \alpha + \beta_{LEI,INV}M_t + \beta_{\mathcal{P}}\mathcal{P}_t + \epsilon_{t+1}$$

	$r_{t+1}^2$			$\bar{r}_{t+1}$		
$LEI_t$	0.1053* (0.0595)	0.1112* (0.0569)	0.1070* (0.0557)	0.0985** (0.0467)	0.1067** (0.0462)	0.1041** (0.0451)
$INV_t$	-0.008** (0.004)	-0.008** (0.003)	1.177* (0.605)	-0.005 (0.003)	-0.006* (0.003)	0.571 (0.531)
Spanned Factors $\mathcal{P}_t$ :	$PC^{1,2}$	$PC^{1-5}$	$f^{1-12}$	$PC^{1,2}$	$PC^{1-5}$	$f^{1-12}$
$T$	341	341	341	341	341	341
Adj. $R^2(\mathcal{P}_t)$	0.4%	0.7%	4.6%	0.2%	0.1%	3.9%
Adj. $R^2(\mathcal{P}_t + M_t)$	1.4%	1.7%	5.4%	1.6%	1.6%	5.2%
$F$ -ratio	2.1	2.1	1.9	2.9*	3.0*	2.8*

Table 8: Maximum likelihood (ML) estimates of the macro-finance model for Nymex crude oil futures, using data from 1/1986 to 6/2014.  $s$ ,  $c$  are the spot price and annualized cost of carry respectively.  $ADS$  and  $LEI$  are the Aruoba-Diebold-Scotti index and the Conference Board Leading Economic Index respectively.  $INV$  is the log of the private U.S. crude oil inventory as reported by the EIA. The coefficients are over a monthly horizon, and the state variables are de-meanded. ML standard errors are in parentheses.

Panel A: Aruoba-Diebold-Scotti (ADS) Index

	$K_1^{\mathbb{P}}$		
	$s_t$	$c_t$	$ADS_t$
$\Delta s_{t+1}$	-0.004 (0.008)	0.059** (0.027)	0.031*** (0.009)
$\Delta c_{t+1}$	0.014* (0.008)	-0.127*** (0.025)	-0.019** (0.009)
$\Delta ADS_{t+1}$	-0.069** (0.033)	0.079 (0.107)	-0.264*** (0.036)

Panel B: Conference Board Leading Economic Index (LEI)

	$K_1^{\mathbb{P}}$		
	$s_t$	$c_t$	$LEI_t$
$\Delta s_{t+1}$	-0.028** (0.011)	0.061** (0.027)	0.126** (0.054)
$\Delta c_{t+1}$	0.028*** (0.010)	-0.128*** (0.025)	-0.074 (0.050)
$\Delta LEI_{t+1}$	-0.002** (0.001)	-0.001 (0.002)	0.003 (0.003)



## A.2 Excluding the Financial Crisis

Inspecting Figure 1, we question whether the results in the paper are driven by the huge swings in oil prices and real activity during 2008-2009. Table 9 presents the forecasting regressions estimated on a subsample from January 1986 to December 2007. We see that the conclusions are the same, and indeed the forecasting power of  $GRO$  is slightly stronger, when we omit 2008-2014.

Table 10 presents the full model estimated on the subsample from January 1986 to December 2007. The subsample estimate is similar to the full-sample estimate, and the key coefficients of  $\Delta GRO_{t+1}$  on  $s_t$  and  $\Delta s_{t+1}$  on  $GRO_t$  remain statistically significant.

## A.3 Time Varying Volatility

This section examines the results of the forecasting regressions in Table 1 when I add measures of time-varying volatility in oil futures. If volatility drives a higher hedge premium, then it might be an omitted factor that explains the positive association between real activity and the oil price forecast. I examine three standard volatility measures:  $optvol_t$  is the implied volatility from short-term options on oil futures,  $garchvol_t$  is the conditional volatility of  $\Delta f_{t+1}^1$  estimated as a GARCH(1,1) process, and  $sqchg_t$  is the lagged squared change  $(\Delta f_t^1)^2$  of the nearby log futures price.

Table 11 shows that the crude oil volatility indexes are indeed negatively correlated with  $GRO$ . However, time-varying volatility does not forecast oil prices or returns, and thus does not explain the forecasting power of real activity. Table 12 shows that none of the volatility factors is significant in the forecasting regressions, none of them significantly raises the adjusted  $R^2$ , and (most importantly) their inclusion does not alter the forecasting power of real activity.

Table 9: Panel A shows the results of forecasting returns to oil futures.  $r_{t+1}^2$  is the log return to the second nearby oil futures contract and  $\bar{r}_{t+1}$  is the average log return to all active futures contracts with maturities from 2 to 12 months. Panel B shows the results of forecasting changes in the level factor  $PC^1$ . The forecasting variables are 1) three sets of 'reduced-form' state variables  $\mathcal{P}_t$  based on oil futures prices and 2) the real activity index  $GRO_t$  and log oil inventory  $INV_t$ . The data are monthly from from 1/1986 to 12/2007. Newey-West standard errors with six lags are in parentheses.

Panel A: Forecasting Futures Returns

$$r_{t+1} = \alpha + \beta_{GRO,INV}M_t + \beta_{\mathcal{P}}\mathcal{P}_t + \epsilon_{t+1}$$

	$r_{t+1}^2$			$\bar{r}_{t+1}$		
$GRO_t$	0.0300*** (0.0089)	0.0281*** (0.0092)	0.0249** (0.0096)	0.0248*** (0.0067)	0.0237*** (0.0070)	0.0212*** (0.0072)
$INV_t$	-0.026 (0.120)	-0.022 (0.115)	-0.018 (0.104)	-0.007 (0.087)	-0.015 (0.080)	-0.012 (0.074)
Spanned Factors $\mathcal{P}_t$ :	$PC^{1,2}$	$PC^{1-5}$	$f^{1-12}$	$PC^{1,2}$	$PC^{1-5}$	$f^{1-12}$
$T$	263	263	263	263	263	263
Adj. $R^2(\mathcal{P}_t)$	-0.5%	-0.2%	5.6%	-0.3%	-0.1%	6.1%
Adj. $R^2(\mathcal{P}_t + M_t)$	2.3%	2.0%	7.1%	2.8%	2.6%	8.1%
$F$ -ratio	4.7***	3.9**	3.1**	5.2***	4.6**	3.7**

Panel B: Forecasting PCs

$$\Delta PC_{t+1} = \alpha + \beta_{GRO,INV}M_t + \beta_{\mathcal{P}}\mathcal{P}_t + \epsilon_{t+1}$$

	$\Delta PC^1$ (Change in level)			$\Delta PC^2$ (Change in slope)		
$GRO_t$	0.0845*** (0.0227)	0.0810*** (0.0236)	0.0729*** (0.0243)	0.0067 (0.0057)	0.0067 (0.0057)	0.0056 (0.0060)
$INV_t$	-0.014 (0.296)	-0.051 (0.270)	-0.039 (0.248)	-0.002 (0.092)	0.004 (0.084)	0.001 (0.078)
Spanned Factors $\mathcal{P}_t$ :	$PC^{1,2}$	$PC^{1-5}$	$f^{1-12}$	$PC^{1,2}$	$PC^{1-5}$	$f^{1-12}$
$T$	263	263	263	263	263	263
Adjusted $R^2(\mathcal{P}_t)$	-0.5%	-0.3%	5.6%	7.4%	8.5%	10.8%
Adj. $R^2(\mathcal{P}_t + M_t)$	2.7%	2.6%	7.7%	7.1%	8.3%	10.4%
$F$ -ratio	5.2***	4.8***	3.8**	0.8	0.7	0.5

Table 10: Maximum likelihood (ML) estimate of the macro-finance model for Nymex crude oil futures using data from 1/1986 to 12/2007.  $s$ ,  $c$  are the spot price and annualized cost of carry respectively.  $GRO$  is the monthly Chicago Fed National Activity Index.  $INV$  is the log of the private U.S. crude oil inventory as reported by the EIA. The coefficients are over a monthly horizon, and the state variables are de-meaned. ML standard errors are in parentheses.

	$K_0^P$	$K_1^P$			
		$s_t$	$c_t$	$GRO_t$	$INV_t$
$s_{t+1}$	0.011 (0.007)	-0.005 (0.014)	0.058 (0.036)	0.029*** (0.011)	-0.005 (0.106)
$c_{t+1}$	-0.012 (0.007)	0.026* (0.014)	-0.130*** (0.036)	-0.012 (0.011)	0.003 (0.108)
$GRO_{t+1}$	0.057 (0.035)	-0.181** (0.070)	0.651*** (0.178)	-0.580*** (0.054)	-1.653 (0.526)
$INV_{t+1}$	0.003 (0.002)	-0.007* (0.004)	0.033*** (0.009)	-0.003 (0.003)	-0.125*** (0.027)

	$K_0^Q$	$K_1^Q$	
		$s_t$	$c_t$
$s_{t+1}$	-0.003 (0.007)	0.000 (0.005)	0.083*** (0.011)
$c_{t+1}$	0.000 (0.012)	-0.009 (0.014)	-0.113*** (0.031)

Shock Volatilities				
[off-diagonal = % correlations]				
	$s$	$c$	$GRO$	$INV$
$s$	0.102			
$c$	-84%	0.056		
$GRO$	7%	-1%	0.499	
$INV$	-20%	29%	2%	0.024

Table 11: The table shows the correlations of the monthly real activity index  $GRO$  and three indexes of time varying volatility in crude oil prices. The time series are monthly from 1/1989 to 6/2014 and have been demeaned.  $garchvol_t$  is the conditional volatility of  $\Delta f_{t+1}^1$  estimated as a GARCH(1,1) process.  $optvol_t$  is the implied volatility based on the prices of at-the-money options on one month futures.  $sqchg_t$  is the squared change  $(\Delta f_t^1)^2$  of the front-month futures contract last month.

	$GRO_t$	$sqchg_t$	$optvol_t$	$garchvol_t$
$GRO_t$	1			
$sqchg_t$	-24.8%	1		
$optvol_t$	-54.9%	50.7%	1	
$garchvol_t$	-51.8%	27.6%	68.7%	1

## A.4 Year-on-Year Changes

Although futures returns are a stationary process (cf. Figure 5), they may contain slow-moving components i.e. time varying expected returns or regime shifts that are effectively nonstationary over a monthly horizon. Log futures prices  $f_t$  and the principal components portfolios  $\mathcal{P}_t$  that summarize them are themselves nonstationary or very close to it (cf. Figure 1). In this setting, forecasting regressions such as those presented in Table 1 may have poor small-sample properties (Ferson, Sarkissian and Simin, 2003).

To address this concern I rerun the regressions in Table 1 after transforming  $f_t$  and  $PC_t$  into year-on-year changes  $f_t^{YOY} = f_t - f_{t-12}$ ,  $PC_t^{YOY} = PC_t - PC_{t-12}$ . The macro variables  $M_t$  need not be transformed as, first, they are clearly stationary (Figure 1) and, second, year-on-year differencing would eliminate the important variation in  $GRO$  (i.e. at business cycle frequency). Table 13 shows that after removing persistence in the regressors, the incremental forecasting power of real activity for futures returns and changes in the level factor is effectively unchanged.

Table 12: The table shows the results of forecasting returns to oil futures including measures of time-varying volatility. The data are monthly from 1/1986 to 6/2014 except *optvol* which is monthly from 1/1989 to 6/2014.  $r_{t+1}^2$  is the log excess return to the second nearby oil futures contract.  $\bar{r}_{t+1}$  is the average log excess return to all active futures contracts with maturities up to 12 months. The forecasting variables are the Chicago Fed National Activity Index  $GRO_t$ , the first two PCs of log oil futures prices, and three measures of crude oil volatility.  $optvol_t$  is the implied volatility based on the prices of at-the-money options on one month futures.  $garchvol_t$  is the conditional volatility of  $\Delta f_{t+1}^1$  estimated as a GARCH(1,1) process.  $sqchg_t$  is the lagged squared change  $(\Delta f_t^1)^2$  of the log price of the first nearby futures contract. Newey-West standard errors with six lags are in parentheses.

$$r_{t+1} = \alpha + \beta_{GRO}M_t + \beta_{\mathcal{P}}PC_t^{1,2} + \beta_{VOL}VOL_t + \epsilon_{t+1}$$

	$r_{t+1}^2$			$\bar{r}_{t+1}$		
$GRO_t$	0.023** (0.010)	0.027*** (0.009)	0.029** (0.012)	0.019** (0.008)	0.020*** (0.007)	0.023** (0.010)
$optvol_t$	-0.009 (0.018)			-0.004 (0.014)		
$garchvol_t$		-0.167 (0.432)			-0.240 (0.314)	
$sqchg_t$			0.002 (0.008)			0.003 (0.006)
Spanned Factors $\mathcal{P}_t$ :	$PC^{1,2}$	$PC^{1,2}$	$PC^{1,2}$	$PC^{1,2}$	$PC^{1,2}$	$PC^{1,2}$
$T$	295	341	341	295	341	341
Adj. $R^2(\mathcal{P}_t + GRO_t)$	4.1%	4.5%	4.5%	3.5%	3.7%	3.7%
Adj. $R^2(\mathcal{P}_t + GRO_t + VOL_t)$	3.9%	4.3%	4.2%	3.2%	3.7%	3.5%
$F$ -ratio	0.4	0.4	0.1	0.1	1.2	0.3

Table 13: Panel A shows the results of forecasting returns to oil futures after transforming to year-on-year changes.  $r_{t+1}^2$  is the log return to the second nearby oil futures contract and  $\bar{r}_{t+1}$  is the average log return to all active futures contracts with maturities from 2 to 12 months. Panel B shows the results of forecasting changes in the level factor  $PC^1$ . The forecasting variables are 1) three sets of year-on-year changes in 'reduced-form' state variables  $\mathcal{P}_t^{YOY}$  based on oil futures prices and 2) the real activity index  $GRO_t$  and log oil inventory  $INV_t$ . The data are monthly from from 1/1986 to 6/2014. Newey-West standard errors with six lags are in parentheses.

Panel A: Forecasting Futures Returns

$$r_{t+1} = \alpha + \beta_{GRO,INV} M_t + \beta_{\mathcal{P}} \mathcal{P}_t^{YOY} + \epsilon_{t+1}$$

	$r_{t+1}^2$			$\bar{r}_{t+1}$		
$GRO_t$	0.0283*** (0.0109)	0.0277** (0.0112)	0.0271** (0.0114)	0.0196** (0.0094)	0.0197** (0.0098)	0.0200* (0.0099)
$INV_t$	-0.039 (0.079)	-0.048 (0.072)	-0.050 (0.070)	-0.038 (0.062)	-0.046 (0.057)	-0.047 (0.055)
Spanned Factors $\mathcal{P}_t^{YOY}$ :	$PC^{1,2}$	$PC^{1-5}$	$f^{1-12}$	$PC^{1,2}$	$PC^{1-5}$	$f^{1-12}$
$T$	329	329	329	329	329	329
Adj. $R^2(\mathcal{P}_t)$	2.4%	2.2%	1.2%	2.0%	1.4%	-0.2%
Adj. $R^2(\mathcal{P}_t + M_t)$	6.5%	5.9%	4.6%	5.0%	4.4%	2.8%
$F$ -ratio	8.0***	7.4***	6.7***	6.2***	6.0***	5.9***

Panel B: Forecasting PCs

$$\Delta PC_{t+1} = \alpha + \beta_{GRO,INV} M_t + \beta_{\mathcal{P}} \mathcal{P}_t^{YOY} + \epsilon_{t+1}$$

	$\Delta PC^1$ (Change in level)			$\Delta PC^2$ (Change in slope)		
$GRO_t$	0.095** (0.043)	0.091** (0.045)	0.088* (0.046)	0.010 (0.006)	0.009 (0.007)	0.008 (0.007)
$INV_t$	-0.231 (0.281)	-0.270 (0.254)	-0.287 (0.251)	0.051 (0.060)	0.034 (0.056)	0.030 (0.055)
Spanned Factors $\mathcal{P}_t$ :	$PC^{1,2}$	$PC^{1-5}$	$f^{1-12}$	$PC^{1,2}$	$PC^{1-5}$	$f^{1-12}$
$T$	329	329	329	329	329	329
Adjusted $R^2(\mathcal{P}_t)$	5.7%	5.7%	7.7%	9.0%	9.5%	11.5%
Adj. $R^2(\mathcal{P}_t + M_t)$	8.4%	8.1%	10.0%	9.4%	9.6%	11.5%
$F$ -ratio	5.8***	5.2***	4.9**	0.8	0.7	0.5

## B Real Option Valuation

I model the log lifting cost (per-barrel cost of extraction) as

$$l_t = \kappa_l + 0.1s_t + 0.01GRO_t + \epsilon_t^l, \quad \epsilon_t^l \sim N(0, \sigma_l)$$

That is,  $l_t$  varies with both  $s_t$  and  $GRO_t$  as well as having an i.i.d. idiosyncratic component with volatility  $\sigma_l$ . The other parameters in the simulated data are in Table 14. Notice the third row of  $K_1^Q$ , which was not present in the model estimates. Pricing assets with payoffs that depend on  $M_t$  requires the risk neutral dynamics of  $M_t$ . In principle one could estimate the risk neutral dynamics of  $M_t$  with a tracking portfolio for  $GRO$  (e.g. Lamont (2001)), but for simplicity I assume that exposure to  $GRO$  carries a fixed risk premium of  $\lambda$ .

I compute option values for different starting values of the lifting cost  $L_0 = \exp(l_0)$ , with  $S_0 = \exp(s_0)$  equal to \$80 per barrel and  $c_0 = 0$ . This simulates an oil firm evaluating wells that differ in their current lifting cost, conditional on a spot price of \$80 and a flat futures curve.

# Internet Appendix

## Macroeconomic Factors in Oil Futures Markets

April 24, 2016



# 1 Model Specification and Risk Premiums

Consider a Gaussian model where the log spot price  $s_t$  of a commodity is a function of  $N_L$  spanned state variables  $L_t$ , which may be latent or observed, and  $N_M$  unspanned state variables  $M_t$  that are observed:

$$\begin{aligned}
 \begin{bmatrix} L_{t+1} \\ M_{t+1} \end{bmatrix} &= K_{0X}^{\mathbb{P}} + K_{1X}^{\mathbb{P}} X_t + \Sigma_X \epsilon_{t+1}^{\mathbb{P}} \\
 L_{t+1} &= K_{0L}^{\mathbb{Q}} + K_{1L}^{\mathbb{Q}} L_t + \Sigma_L \epsilon_{t+1}^{\mathbb{Q}} \\
 s_t &= \delta_0 + \delta_1' L_t
 \end{aligned} \tag{1}$$

where

- $\mathbb{P}$  denotes dynamics under the historical or data generating measure
- $\mathbb{Q}$  denotes dynamics under the risk neutral measure
- $\epsilon_{L,t+1}^{\mathbb{Q}} \sim N(0, I_{N_L})$ ,  $\epsilon_{t+1}^{\mathbb{P}} \sim N(0, I_N)$
- $\Sigma_L$  is the top left  $N_L \times N_L$  block of  $\Sigma_X$ ;  $\Sigma_L, \Sigma_X$  are lower triangular

(1) is equivalent to specifying the equation for  $s_t$  and the  $\mathbb{P}$ -dynamics plus a lognormal affine discount factor with 'essentially affine' prices of risk as in [Duffee \(2002\)](#). For  $N_M = 0$  the framework includes models such as [Gibson and Schwartz \(1990\)](#); [Schwartz \(1997\)](#); [Schwartz and Smith \(2000\)](#) as special cases (see Appendix 4). Standard recursions show that (1) implies affine log prices for futures,

$$f_t = A + BX_t \tag{2}$$

$$f_t = \begin{bmatrix} f_t^1 & f_t^2 & \dots & f_t^J \end{bmatrix}'$$

where  $f_t^j$  is the price of a  $j$  period future and  $J$  is the number of futures with different maturities.

Estimating the model as written presents difficulties; with two latent factors and two macro factors there are 40 free parameters. Different sets of parameter values are observationally equivalent due to rotational indeterminacy of the latent factors. Discussing models of the form (1) for bond yields, [Hamilton and Wu \(2012\)](#) refer to “*tremendous numerical challenges in estimating the necessary parameters from the data due to highly nonlinear and badly behaved likelihood surfaces.*” In general, affine models for futures identify the model by specifying dynamics that are less general than (1).

[Joslin, Priebsch and Singleton \(2014\)](#) note that if  $N_L$  linear combinations of bond yields are measured without error, then any model of yields of the form (1) implies a model with observable factors in place of the latent factors. They construct a minimal parametrization where no sets of parameters are redundant - models in the “JPS form” are unique. Thus the likelihood surface is well behaved and contains a single global maximum. Their results hold to a very close approximation if the linear combinations of yields are observed with relatively small and idiosyncratic errors.

Section 2 demonstrates the same result for futures pricing: if  $N_L$  linear combinations of log futures prices are measured without error,

$$\mathcal{P}_t = W f_t \tag{3}$$

for any full rank  $N_L \times J$  matrix  $W$ , then any model of the form (1) is observationally equivalent to a unique model of the form

$$\begin{aligned}
\begin{bmatrix} \Delta \mathcal{P}_{t+1} \\ \Delta UM_{t+1} \end{bmatrix} &= \Delta Z_{t+1} = K_0^{\mathbb{P}} + K_1^{\mathbb{P}} Z_t + \Sigma_Z \epsilon_{t+1}^{\mathbb{P}} \\
\Delta \mathcal{P}_{t+1} &= K_0^{\mathbb{Q}} + K_1^{\mathbb{Q}} \mathcal{P}_t + \Sigma_{\mathcal{P}} \epsilon_{t+1}^{\mathbb{Q}} \\
s_t &= \rho_0 + \rho_1 \mathcal{P}_t
\end{aligned} \tag{4}$$

parametrized by  $\theta = (\lambda^{\mathbb{Q}}, p_{\infty}, \Sigma_Z, K_0^{\mathbb{P}}, K_1^{\mathbb{P}})$ , where

- $\lambda^{\mathbb{Q}}$  are the  $N_L$  ordered eigenvalues of  $K_1^{\mathbb{Q}}$
- $p_{\infty}$  is a scalar intercept
- $\Sigma_Z$  is the lower triangular Cholesky decomposition of the covariance matrix of innovations in the state variables
- $\Sigma_{\mathcal{P}} \Sigma'_{\mathcal{P}} = [\Sigma_Z \Sigma'_Z]_{N_L}$ , the top left  $N_L \times N_L$  block of  $\Sigma_Z \Sigma'_Z$

## 1.1 $\mathcal{P}_t$ Measured Without Error

In this paper I assume that while each of the log futures maturities is observed with iid measurement error, the pricing factors  $\mathcal{P}_t^1$  and  $\mathcal{P}_t^2$  are measured without error.

$$f_t^j = A_j + B_j \mathcal{P}_t + \nu_t^j, \quad \nu_t^j \sim N(0, \zeta_j^2)$$

The use of the first two PCs of log price levels is not important: in unreported results I find that all estimates and results are effectively identical using other alternatives such as the

first two PCs of log price changes or of returns, or a priori weights such as

$$W = \begin{bmatrix} 1 & \dots & 1 \\ 0 & \dots & 12 \end{bmatrix}$$

The identifying assumption that  $N_L$  linear combinations of yields are measured without error is common in the bond yields literature beginning with [Chen and Scott \(1993\)](#). Given the model parameters, values of the latent factors at each date are then extracted by inverting the relation (2). The same assumption is used to identify previous latent factor models for commodity futures (see [Gibson and Schwartz \(1990\)](#); [Casassus and Collin-Dufresne \(2005\)](#); [Hamilton and Wu \(2014\)](#)). In unreported results I find that all estimates and results are effectively identical if the pricing factors are estimated via the Kalman filter.

## 1.2 Rotating to $s_t$ and $c_t$

Once the model is estimated in the JPS form, I rotate  $(\mathcal{P}_t^1, \mathcal{P}_t^2)$  to be the model implied log spot price and instantaneous cost of carry,  $(s_t, c_t)$ . For  $s_t$  this is immediate:

$$s_t = \rho_0 + \rho_1 \mathcal{P}_t$$

For  $c_t$  the definition is as follows. Any agent with access to a storage technology can buy the spot commodity, sell a one month future, store for one month and make delivery. Add up all the costs and benefits of doing so (including interest, costs of storage, and convenience yield) and express them as quantity  $c_t$  where the total cost in dollar terms =  $S_t(e^{c_t} - 1)$ .

Then in the absence of arbitrage it must be the case that

$$F_t^1 = S_t e^{c_t}$$

$$f_t^1 = s_t + c_t = E_t^{\mathbb{Q}}[s_{t+1}] + \frac{1}{2}\sigma_s^2$$

$$c_t = E_t^{\mathbb{Q}}[\Delta s_{t+1}] + \frac{1}{2}\sigma_s^2$$

$$= \rho_1[K_0^{\mathbb{Q}} + K_1^{\mathbb{Q}}\mathcal{P}_t] + \frac{1}{2}\sigma_s^2$$

### 1.3 Risk premiums and $s_t, c_t$ :

Szymanowska et al. (2013) define the per-period log basis  $y_t^n \equiv f_t^n - s_t$ , and define two risk premiums based on different futures trading strategies; the spot premium  $\pi_{s,t}$  and the term premium  $\pi_{y,t}^n$ .

The spot premium is defined as

$$\begin{aligned} \pi_{s,t} &\equiv E_t[s_{t+1} - s_t] - y_t^1 \\ &= E_t[s_{t+1}] - f_t^1 = E_t^{\mathbb{P}}[s_{t+1}] - E_t^{\mathbb{Q}}[s_{t+1}] - \frac{1}{2}\sigma_s^2 \\ &\Rightarrow \pi_{s,t} = \Lambda_{s_t} - \frac{1}{2}\sigma_s^2 \end{aligned}$$

The term premium is defined as

$$\begin{aligned} \pi_{y,t}^n &\equiv y_t^1 + (n-1)E_t[y_{t+1}^{n-1}] - ny_t^n \\ &= f_t^1 + (n-1)E_t[f_{t+1}^{n-1}] - nf_t^n \end{aligned}$$

The one month term premium is always zero, because storing for one month is riskless.

$$\pi_{y,t}^{(1)} = f_t^1 + 0 - f_t^1 = 0$$

$$\pi_{y,t}^{(2)} = c_t + E_t^{\mathbb{P}} [c_{t+1}] - 2E_t^{\mathbb{Q}} [s_{t+2} - s_{t+1} + s_{t+1} - s_t]$$

$$= E_t^{\mathbb{P}} [c_{t+1}] - E_t^{\mathbb{Q}} [c_{t+1}] - E_t^{\mathbb{Q}} [s_{t+2} - s_{t+1}] - c_t$$

$$\Rightarrow \pi_{y,t}^{(2)} = \Lambda_{c_t} - (c_t + E_t^{\mathbb{Q}} [c_{t+1}])$$

Thus the spot premium and term premium of [Szymanowska et al. \(2013\)](#) each have a natural expression in our affine framework. The spot premium is exactly the risk premium attached to shocks to the log spot price  $s_t$  plus a small constant. The term premium is the risk premium attached to shocks to the cost of carry minus the (risk-neutral) total expected cost of carry.

## 2 JPS Parametrization

I assume that  $N_L$  linear combinations of log futures prices are measured without error,

$$\mathcal{P}_t^L = W f_t$$

for any full-rank real valued  $N_L \times J$  matrix  $W$ , and show that any model of the form

$$\begin{aligned} \begin{bmatrix} \Delta L_{t+1} \\ \Delta M_{t+1} \end{bmatrix} &= \Delta X_{t+1} = K_{0X}^{\mathbb{P}} + K_{1X}^{\mathbb{P}} X_t + \Sigma_X \epsilon_{t+1}^{\mathbb{P}} \\ \Delta L_{t+1} &= K_{0L}^{\mathbb{Q}} + K_{1L}^{\mathbb{Q}} X_t + \Sigma_L \epsilon_{L,t+1}^{\mathbb{Q}} \\ s_t &= \delta_0 + \delta_1' X_t \end{aligned} \tag{5}$$

is observationally equivalent to a unique model of the form

$$\begin{aligned} \begin{bmatrix} \Delta \mathcal{P}_{t+1}^L \\ \Delta M_{t+1} \end{bmatrix} &= \Delta Z_{t+1} = K_0^{\mathbb{P}} + K_1^{\mathbb{P}} Z_t + \Sigma_Z \epsilon_{Z,t+1}^{\mathbb{P}} \\ \Delta \mathcal{P}_{t+1}^L &= K_0^{\mathbb{Q}} + K_1^{\mathbb{Q}} Z_t + \Sigma_{\mathcal{P}} \epsilon_{t+1}^{\mathbb{Q}} \\ s_t &= \rho_0 + \rho_1' Z_t \end{aligned} \tag{6}$$

which is parametrized by  $\theta = (\lambda^{\mathbb{Q}}, p_{\infty}, \Sigma_Z, K_0^{\mathbb{P}}, K_1^{\mathbb{P}})$ .

The proof that follows is essentially the same as that of [Joslin, Priebsch and Singleton \(2014\)](#). [Joslin, Singleton and Zhu \(2011\)](#) demonstrates the result for all cases including zero, repeated and complex eigenvalues.

Assume the model (5) under consideration is nonredundant, that is, there is no observationally equivalent model with fewer than  $N$  state variables. If there is such a model, switch to it and proceed.

## 2.1 Observational Equivalence

Given any model of the form (5), the  $J \times 1$  vector of log futures prices  $f_t$  is affine in  $L_t$ ,

$$f_t = A_L + B_L L_t$$

Hence the set of  $N_L$  linear combinations of futures prices,  $\mathcal{P}_t^L$ , is as well:

$$\mathcal{P}_t^L = W_L f_t = W_L A_L + W_L B_L L_t$$

Assume that the  $N_L$  ordered elements of  $\lambda^{\mathbb{Q}}$ , the eigenvalues of  $K_{1L}^{\mathbb{Q}}$ , are real, distinct and nonzero. There exists a matrix  $C$  such that  $K_{1L}^{\mathbb{Q}} = C \text{diag}(\lambda^{\mathbb{Q}}) C^{-1}$ . Define  $D = C \text{diag}(\delta_1) C^{-1}$ ,  $D^{-1} = C \text{diag}(\delta_1)^{-1} C^{-1}$  and

$$\begin{aligned} Y_t &= D[L_t + (K_{1L}^{\mathbb{Q}})^{-1} K_{0L}^{\mathbb{Q}}] \\ \Rightarrow L_t &= D^{-1} Y_t - (K_{1L}^{\mathbb{Q}})^{-1} K_{0L}^{\mathbb{Q}} \end{aligned}$$

Then

$$\begin{aligned} \Delta Y_{t+1} &= D \Delta L_{t+1} \\ &= D[K_{0L}^{\mathbb{Q}} + K_{1L}^{\mathbb{Q}}(D^{-1} Y_t - (K_{1L}^{\mathbb{Q}})^{-1} K_{0L}^{\mathbb{Q}}) + \Sigma_L \epsilon_{L,t+1}^{\mathbb{Q}}] \\ &= \text{diag}(\lambda^{\mathbb{Q}}) Y_t + D \Sigma_L \epsilon_{L,t+1}^{\mathbb{Q}} \end{aligned}$$



and

$$\begin{aligned} \begin{bmatrix} \Delta Y_{t+1} \\ \Delta M_{t+1} \end{bmatrix} &= \begin{bmatrix} D & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} [K_{0X}^{\mathbb{P}} + K_{1X}^{\mathbb{P}} \left( \begin{bmatrix} D^{-1} & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \begin{bmatrix} Y_t \\ M_t \end{bmatrix} - \begin{bmatrix} (K_{1L}^{\mathbb{Q}})^{-1} K_{0L}^{\mathbb{Q}} \\ 0 \end{bmatrix} \right) + \Sigma_X \epsilon_{t+1}^{\mathbb{P}}] \\ &= K_{0Y}^{\mathbb{P}} + K_{1Y}^{\mathbb{P}} \begin{bmatrix} Y_t \\ M_t \end{bmatrix} + \begin{bmatrix} D & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \Sigma_X \epsilon_{t+1}^{\mathbb{P}} \end{aligned}$$

and

$$p_t = \delta_0 + \delta_1' L_t = \delta_0 + \delta_1' D^{-1} Y_t - \delta_1' (K_{1L}^{\mathbb{Q}})^{-1} K_{0L}^{\mathbb{Q}} = p_{\infty} + \iota \cdot Y_t$$

where  $\iota$  is a row of  $N_L$  ones.

$$f_t = A_Y + B_Y Y_t$$

$$\mathcal{P}_t^L = W f_t = W A_Y + W B_Y Y_t$$

The model is nonredundant  $\Rightarrow W B_Y$  is invertible:

$$Y_t = (W B_Y)^{-1} \mathcal{P}_t^L - (W B_Y)^{-1} W A_Y$$

$$\begin{aligned} \cdot \mathcal{P}_{t+1}^L &= W B_Y \Delta Y_{t+1} = W B_Y \text{diag}(\lambda^{\mathbb{Q}}) [(W B_Y)^{-1} \mathcal{P}_t^L - (W B_Y)^{-1} W A_Y] + W B_Y D \Sigma_L \epsilon_{L,t+1}^{\mathbb{Q}} \\ &= K_0^{\mathbb{Q}} + K_1^{\mathbb{Q}} \mathcal{P}_t^L + \Sigma_{\mathcal{P}} \epsilon_{t+1}^{\mathbb{Q}} \end{aligned}$$

Further,

$$\begin{aligned}
\Delta Z_{t+1} &= \begin{bmatrix} \cdot \mathcal{P}_{t+1}^L \\ \Delta M_{t+1} \end{bmatrix} = \begin{bmatrix} WB_Y & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \begin{bmatrix} \Delta Y_{t+1} \\ \Delta M_{t+1} \end{bmatrix} \\
&= \begin{bmatrix} WB_Y & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \left( K_{0Y}^{\mathbb{P}} + K_{1Y}^{\mathbb{P}} \begin{bmatrix} Y_t \\ M_t \end{bmatrix} + \begin{bmatrix} D & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \Sigma_X \epsilon_{t+1}^{\mathbb{P}} \right) \\
&= K_0^{\mathbb{P}} + K_1^{\mathbb{P}} Z_t + \Sigma_Z \epsilon_{t+1}^{\mathbb{P}}
\end{aligned}$$

$$p_t = p_{\infty} + \iota \cdot Y_t = p_{\infty} + \iota \cdot (WB_Y)^{-1} \mathcal{P}_t^L - \iota \cdot (WB_Y)^{-1} W A_Y = \rho_0 + \rho_1' \mathcal{P}_t^L$$

QED. Collecting the formulas: given any model of the form (1), there is an observationally equivalent model of the form (4), parametrized by  $\theta = (\lambda^{\mathbb{Q}}, p_{\infty}, \Sigma_Z, K_0^{\mathbb{P}}, K_1^{\mathbb{P}})$ , where

- $D = C \text{diag}(\delta_1)^{-1} C^{-1}$
- $\Sigma_Z = \begin{bmatrix} WB_Y D & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \Sigma_X, \Sigma_{\mathcal{P}} = [\Sigma_Z]_{\mathcal{L}\mathcal{L}}$
- $B_Y = \begin{bmatrix} \iota' [I_{\mathcal{L}+\mathcal{M}} + \text{diag}(\lambda^{\mathbb{Q}})] \\ \vdots \\ \iota' [I_{\mathcal{L}+\mathcal{M}} + \text{diag}(\lambda^{\mathbb{Q}})]^J \end{bmatrix}$
- $A_Y = \begin{bmatrix} p_{\infty} + \frac{1}{2} \iota' \Sigma_{\mathcal{P}} \Sigma_{\mathcal{P}}' \iota \\ \vdots \\ A_{Y,J-1} + \frac{1}{2} B_{Y,J-1} \Sigma_{\mathcal{P}} \Sigma_{\mathcal{P}}' B_{Y,J-1}' \end{bmatrix}$
- $K_1^{\mathbb{Q}} = WB_Y \text{diag}(\lambda^{\mathbb{Q}}) (WB_Y)^{-1}, K_0^{\mathbb{Q}} = -K_1^{\mathbb{Q}} W A_Y$
- $\rho_0 = p_{\infty} - \iota \cdot (WB_Y)^{-1} W A_Y, \rho_1' = \iota \cdot (WB_Y)^{-1}$

In estimation I adopt the alternate form

- $\Delta Y_{t+1} = \begin{bmatrix} p_\infty \\ 0 \end{bmatrix} + \text{diag}(\lambda^\mathbb{Q})Y_t + D\Sigma_X\epsilon_{t+1}^\mathbb{Q}$
- $p_t = \iota \cdot Y_t$
- $A_Y = \begin{bmatrix} p_\infty + \frac{1}{2}\iota'\Sigma_{\mathcal{P}}\Sigma'_{\mathcal{P}}\iota \\ \vdots \\ A_{Y,J-1} + B_{Y,J-1} \begin{bmatrix} p_\infty \\ 0 \end{bmatrix} + \frac{1}{2}B_{Y,J-1}\Sigma_{\mathcal{P}}\Sigma'_{\mathcal{P}}B'_{Y,J-1} \end{bmatrix}$
- $K_1^\mathbb{Q} = WB_Y\text{diag}(\lambda^\mathbb{Q})(WB_Y)^{-1}$ ,  $K_0^\mathbb{Q} = WB_Y \begin{bmatrix} p_\infty \\ 0 \end{bmatrix} - K_1^\mathbb{Q}WA_Y$
- $\rho_0 = -\iota \cdot (WB_Y)^{-1}WA_Y$ ,  $\rho'_1 = \iota \cdot (WB_Y)^{-1}$

which is numerically stable when  $\lambda^\mathbb{Q}(1) \rightarrow 0$ . See the online supplement to [JSZ 2011](#).

## 2.2 Uniqueness

We consider two models of the form (4) with parameters  $\theta$  and  $\hat{\theta} = (\hat{\lambda}^\mathbb{Q}, \hat{p}_\infty, \hat{\Sigma}_Z, \hat{K}_0^\mathbb{P}, \hat{K}_1^\mathbb{P})$  that are observationally equivalent and show that this implies  $\theta = \hat{\theta}$ .

Since  $Z_t = \begin{bmatrix} \mathcal{P}_t^L \\ M_t \end{bmatrix}$  are all observed,  $\{\Sigma_Z, K_0^\mathbb{P}, K_1^\mathbb{P}\} = \{\hat{\Sigma}_Z, \hat{K}_0^\mathbb{P}, \hat{K}_1^\mathbb{P}\}$ .

Since  $f_t = A + BZ_t$  are observed,  $A(\theta) = A(\hat{\theta})$ ,  $B(\theta) = B(\hat{\theta})$ .

Suppose  $\lambda^\mathbb{Q} \neq \hat{\lambda}^\mathbb{Q}$ . Then by the uniqueness of the ordered eigenvalue decomposition,

$$B_Y^j(\lambda) \neq B_Y^j(\hat{\lambda}) \forall j$$

$$\Rightarrow WB_Y(\lambda) \neq WB_Y(\hat{\lambda}) \Rightarrow (WB_Y(\lambda))^{-1} \neq (WB_Y(\hat{\lambda}))^{-1}$$

$$\Rightarrow \rho_1(\lambda) \neq \rho_1(\hat{\lambda}) \Rightarrow B(\lambda) \neq B(\hat{\lambda})$$

, a contradiction. Hence  $\lambda^{\mathbb{Q}} = \hat{\lambda}^{\mathbb{Q}}$ . Then  $A(\lambda^{\mathbb{Q}}, p^{\infty}) = A(\hat{\lambda}^{\mathbb{Q}}, \hat{p}^{\infty}) \Rightarrow p^{\infty} = \hat{p}^{\infty}$ .

### 3 Estimation

Given the futures prices and macroeconomic time series  $\{f_t, M_t\}_{t=1, \dots, T}$  and the set of portfolio weights  $W$  that define the pricing factors:

$$\mathcal{P}_t = W f_t$$

we need to estimate the minimal parameters  $\theta = (\lambda^{\mathbb{Q}}, p_{\infty}, \Sigma_Z, K_0^{\mathbb{P}}, K_1^{\mathbb{P}})$  in the JPS form. The estimation is carried out by maximum likelihood (ML). If no restrictions are imposed (i.e. we are estimating the canonical model (5)), then  $K_0^{\mathbb{P}}, K_1^{\mathbb{P}}$  do not affect futures pricing and are estimated consistently via OLS. Otherwise  $K_0^{\mathbb{P}}, K_1^{\mathbb{P}}$  are obtained by GLS taking the restrictions into account. The OLS estimate of  $\Sigma_Z$  is used as a starting value, and the starting value for  $p_{\infty}$  is the unconditional average of the nearest-maturity log futures price. Both were always close to their ML value. Finally I search over a range of values for the eigenvalues  $\lambda^{\mathbb{Q}}$ .

After the ML estimate of the model in the JPS form is found, I rotate and translate the spanned factors from  $\mathcal{P}_t^1, \mathcal{P}_t^2$  to  $s_t, c_t$  as described in 1.2. I rotate and translate  $UM_t$  to  $M_t$ , so that the estimate reflects the behavior of the time series  $M_t$ :

$$\begin{bmatrix} s_t \\ c_t \\ M_t \end{bmatrix} = \begin{bmatrix} \rho_0 \\ \frac{1}{2}\sigma_s^2 + \rho_1 K_0^{\mathbb{Q}} \\ \alpha_{M\mathcal{P}} \end{bmatrix} + \begin{bmatrix} \rho_1 & 0_{1 \times N_M} \\ \rho_1 K_1^{\mathbb{Q}} & 0_{1 \times N_M} \\ 0_{N_M \times 1} & \beta_{M\mathcal{P}} \end{bmatrix} \begin{bmatrix} \mathcal{P}_t \\ UM_t \end{bmatrix}$$

where

$$M_t = \alpha_{MP} + \beta_{MP}\mathcal{P}_t + UM_t$$

## 4 Comparison with other Futures Models

The model (1) is a canonical affine Gaussian model, so any affine Gaussian model is a special case. For example, the [Gibson and Schwartz \(1990\)](#); [Schwartz \(1997\)](#); [Schwartz and Smith \(2000\)](#) two factor model in discrete time is the following:

$$\begin{bmatrix} \Delta s_{t+1} \\ \Delta \delta_{t+1} \end{bmatrix} = \begin{bmatrix} \mu \\ \kappa\alpha \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & -\kappa \end{bmatrix} \begin{bmatrix} s_t \\ \delta_t \end{bmatrix} + \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}^{1/2} \epsilon_{t+1}^{\mathbb{P}} \quad (7)$$

$$\begin{bmatrix} \Delta s_{t+1} \\ \Delta \delta_{t+1} \end{bmatrix} = \begin{bmatrix} r \\ \kappa\alpha - \lambda \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & -\kappa \end{bmatrix} \begin{bmatrix} s_t \\ \delta_t \end{bmatrix} + \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}^{1/2} \epsilon_{t+1}^{\mathbb{Q}} \quad (8)$$

which is clearly a special case of (1).

The [Casassus and Collin-Dufresne \(2005\)](#) model in discrete time is:

$$\begin{bmatrix} \Delta X_{t+1} \\ \Delta \hat{\delta}_{t+1} \\ \Delta r_{t+1} \end{bmatrix} = \begin{bmatrix} \kappa_X^P \theta_X^P + \kappa_{Xr}^P \theta_r^P + \kappa_{X\delta}^P \theta_\delta^P \\ \kappa_\delta^P \theta_\delta^P \\ \kappa_r^P \theta_r^P \end{bmatrix} + \begin{bmatrix} -\kappa_X^P & -\kappa_{X\delta}^P & -\kappa_{Xr}^P \\ 0 & -\kappa_\delta^P & 0 \\ 0 & 0 & -\kappa_r^P \end{bmatrix} \begin{bmatrix} X_t \\ \hat{\delta}_t \\ r_t \end{bmatrix} + \begin{bmatrix} \sigma_X & 0 & 0 \\ 0 & \sigma_\delta & 0 \\ 0 & 0 & \sigma_r \end{bmatrix} \begin{bmatrix} 1 \\ \rho_{X\delta} & 1 \\ \rho_{Xr} & \rho_{\delta r} & 1 \end{bmatrix}^{1/2} \epsilon_{t+1}^{\mathbb{P}} \quad (9)$$

$$\begin{bmatrix} \Delta X_{t+1} \\ \Delta \hat{\delta}_{t+1} \\ \Delta r_{t+1} \end{bmatrix} = \begin{bmatrix} \alpha_X \theta_X^Q + (\alpha_r - 1) \theta_r^Q + \theta_\delta^Q \\ \kappa_\delta^Q \theta_\delta^Q \\ \kappa_r^Q \theta_r^Q \end{bmatrix} + \begin{bmatrix} -\alpha_X & -1 & 1 - \alpha_r \\ 0 & -\kappa_\delta^Q & 0 \\ 0 & 0 & -\kappa_r^Q \end{bmatrix} \begin{bmatrix} X_t \\ \hat{\delta}_t \\ r_t \end{bmatrix} + \begin{bmatrix} \sigma_X & 0 & 0 \\ 0 & \sigma_\delta & 0 \\ 0 & 0 & \sigma_r \end{bmatrix} \begin{bmatrix} 1 \\ \rho_{X\delta} & 1 \\ \rho_{Xr} & \rho_{\delta r} & 1 \end{bmatrix}^{1/2} \epsilon_{t+1}^{\mathbb{Q}} \quad (10)$$

(see their formulas 7, 12, 13 and 27, 28, 30) which is the Schwartz three factor model with more flexible risk premiums.

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Table 14: Parameters of the calibration for computing real option values

	$K_0^{\mathbb{P}}$	$K_1^{\mathbb{P}}$			$s$	$\Sigma$		
		$s_t$	$c_t$	$GRO_t$		$s$	$c$	$GRO$
						$c$		
$s_{t+1}$	0.00	1.00	0.083	0.03	$GRO$	0.10	-0.08	0.06
$c_{t+1}$	0.00	0.00	0.90	0.00		0	0	0.50
$GRO_{t+1}$	0.00	-0.10	0.00	0.60				
	$K_0^{\mathbb{Q}}$	$K_1^{\mathbb{Q}}$						
		$s_t$	$c_t$	$GRO_t$				
$s_{t+1}$	0.00	1.00	0.083	0.00				
$c_{t+1}$	0.00	0.00	0.90	0.00				
$GRO_{t+1}$	$-\lambda$	-0.10	0.00	0.60				